DEPARTMENT OF EDUCATION

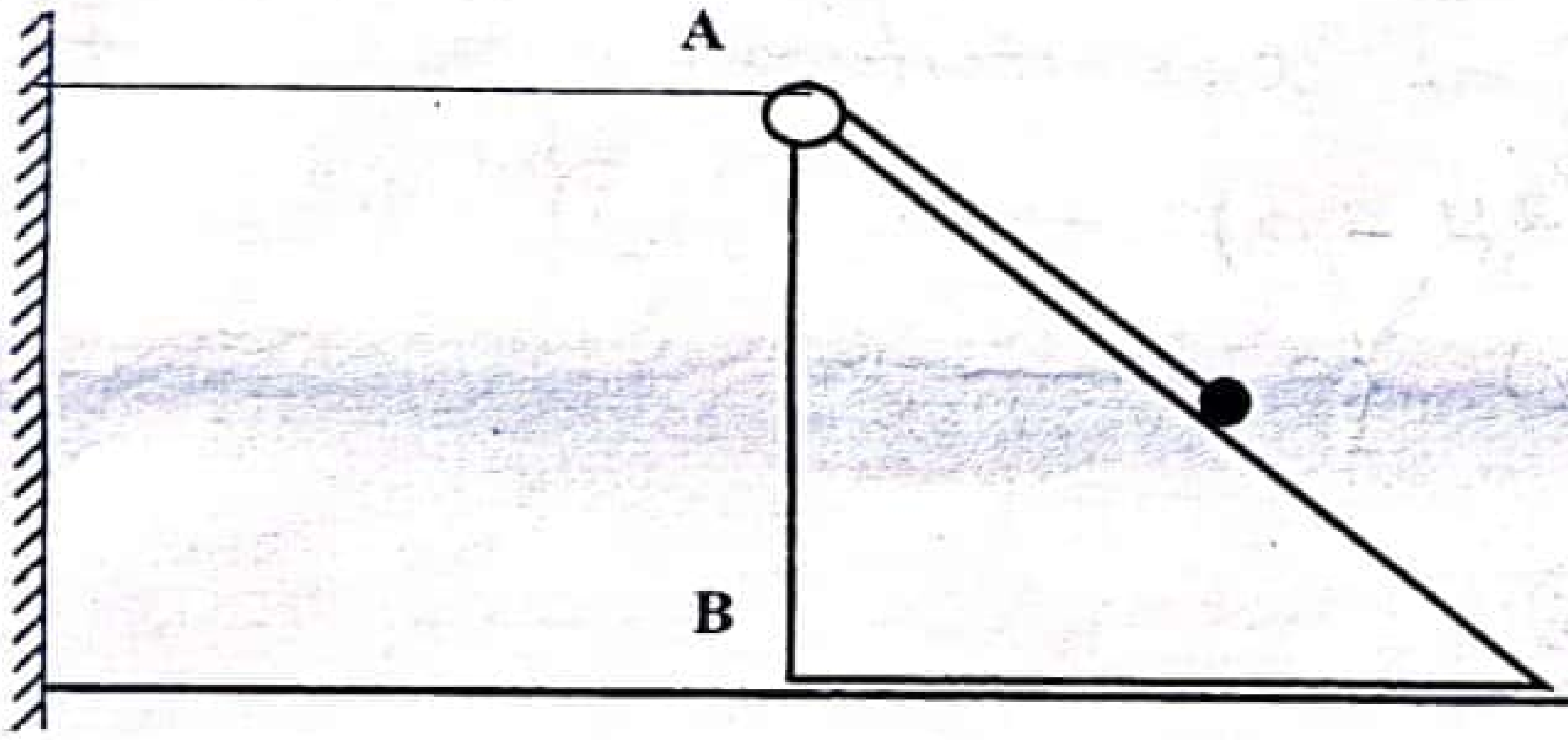
CENTRAL PROVINCE

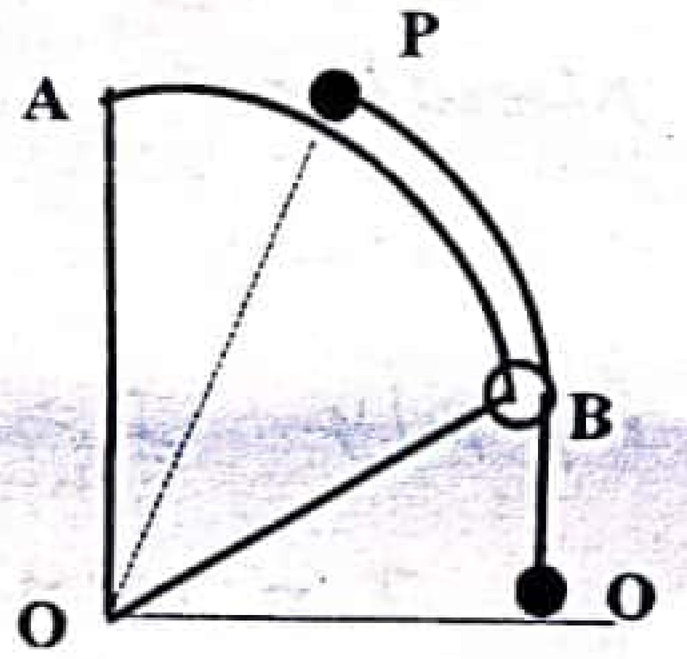
G.C.E.(A/L) Examination – 2023

Combined Mathematics II

Marking Scheme

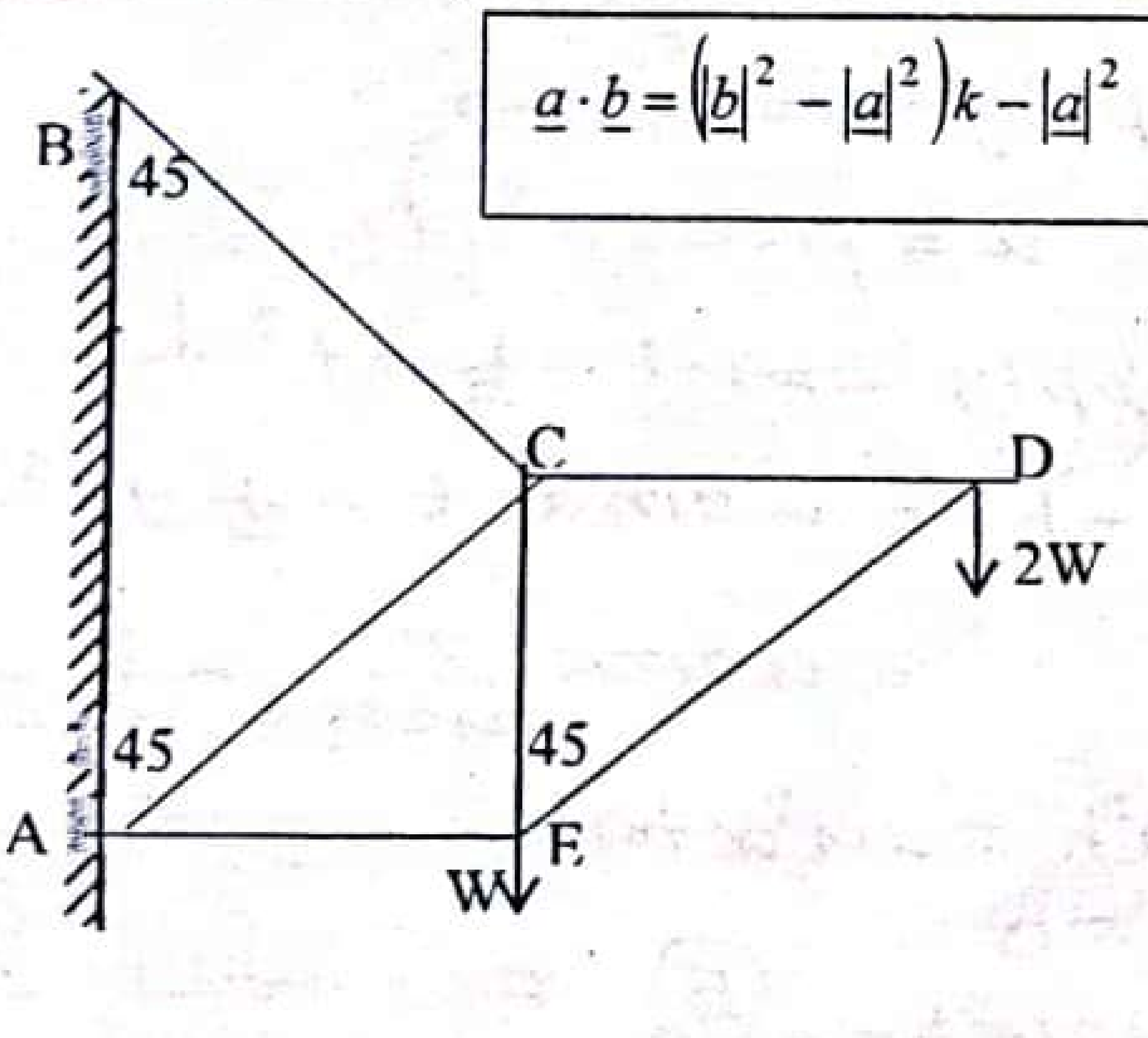
Grade 13

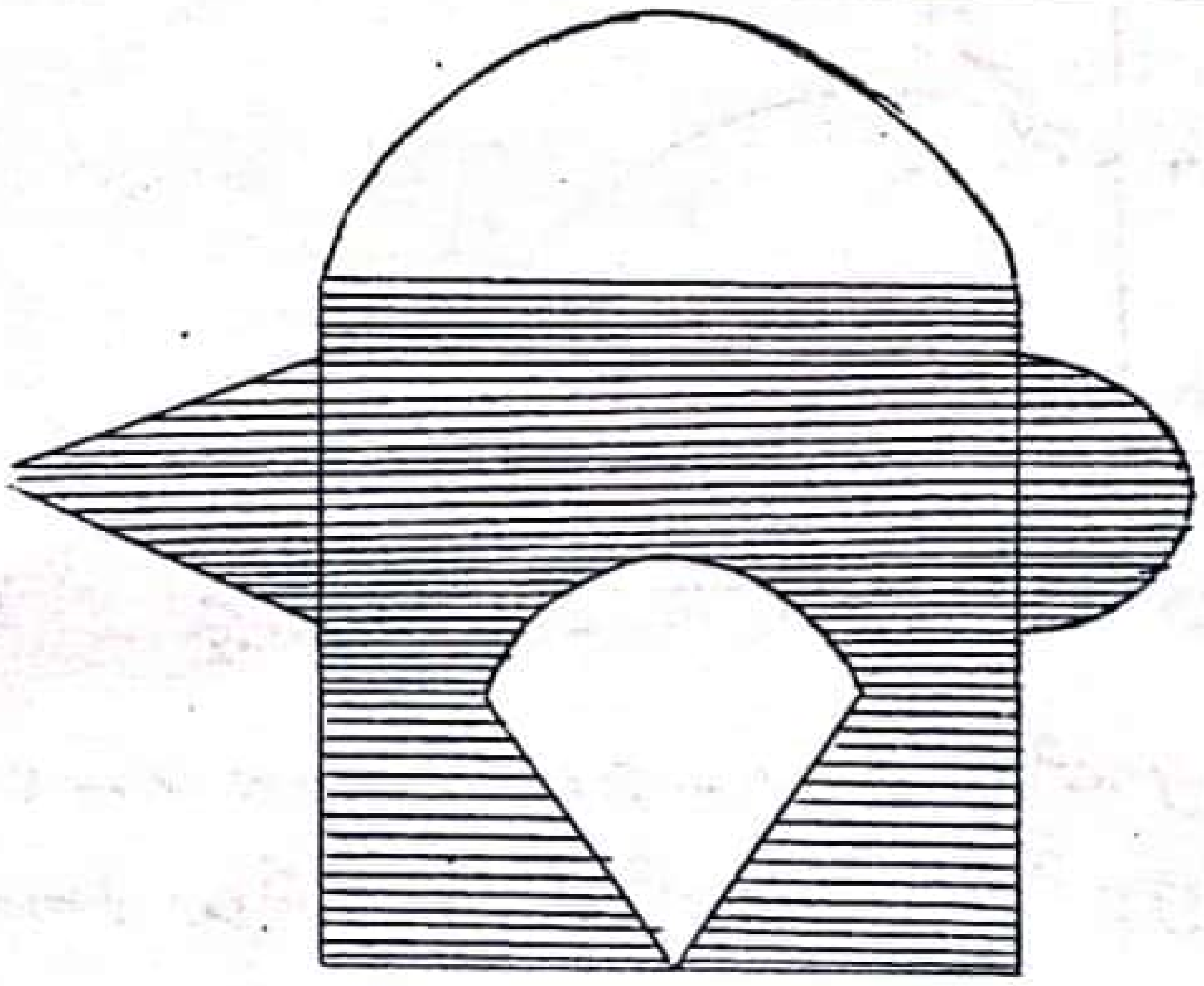




$$\frac{w}{2}\sqrt{(2\lambda+1)^2\tan^2\alpha+4\lambda^2}$$

$$\underline{a}\cdot\underline{b}=\left(|\underline{b}|^2-|\underline{a}|^2\right)k-|\underline{a}|^2$$

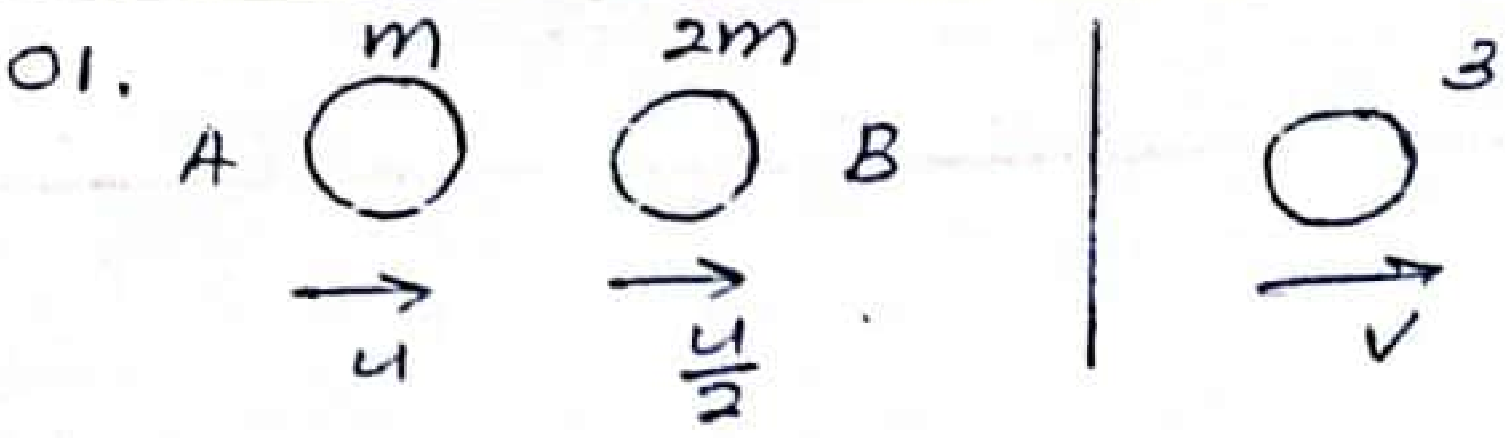




$$a\dot{\theta}^2=\frac{2g}{3}\left[\cos\alpha-\cos\theta+2\theta-2\alpha\right]$$

$$\sqrt{\frac{\ell}{g}}\left[1+\frac{1}{\sqrt{2}}\left(\pi-\cos^{-1}\frac{1}{\sqrt{3}}\right)\right]$$

01.



Apply the principle of linear momentum

$$mu + 2m\frac{u}{2} = 3mV$$

$$V = \frac{2}{3}u \quad ⑤$$

$e = \frac{1}{2}$

Apply $\underline{I} = \Delta(m\underline{v})$ to the whole system →

$$0 = (3mx + my) - 3m.\frac{2u}{3} \quad ⑤$$

$$3x + y = 2u \quad — ①$$

$x = \frac{5u}{12}$ ⑤

$y = \frac{3u}{4}$

Newton's Law of Restitution

$$x - y = -\frac{1}{2}\left(\frac{2u}{3} - 0\right) \quad — ② \quad ⑤$$

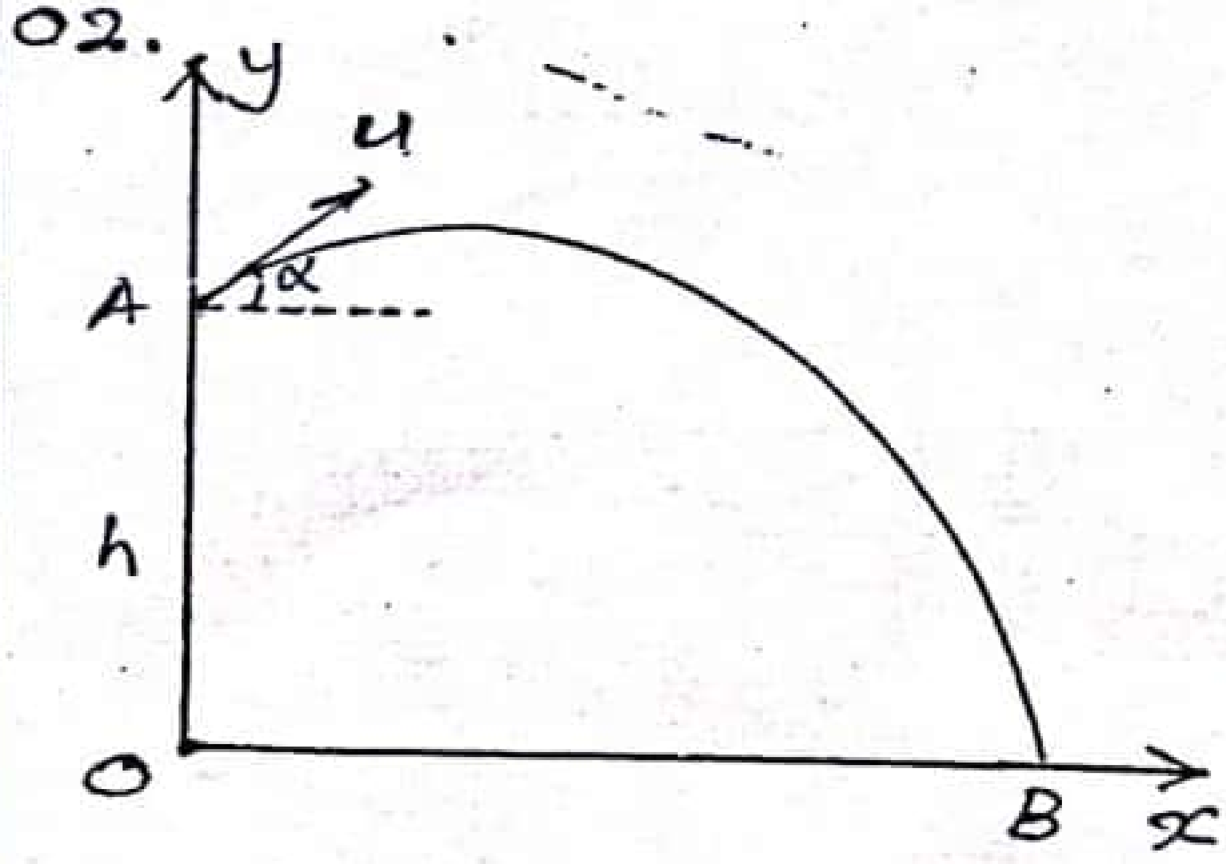
Apply $\underline{I} = \Delta(m\underline{v})$ to m →

$$I = my \quad ⑤$$

$$= \frac{3mu}{4}$$

$\boxed{25}$

02.



Apply $s = ut$ →

$$x = u\cos\alpha . t \quad ⑤$$

Apply $s = ut + \frac{1}{2}at^2$ ↑

$$-h = u\sin\alpha . t - \frac{1}{2}gt^2 \quad ⑤$$

$$= u\sin\alpha . \frac{x}{u\cos\alpha} - \frac{g}{2}\frac{x^2}{u^2\cos^2}$$

$$-2u^2h = 2u^2x\tan\alpha - gx^2\sec^2\alpha$$

$$gx^2\sec^2\alpha - 2u^2x\tan\alpha - 2u^2h = 0$$

$$gx^2(1+\tan^2\alpha) - 2u^2x\tan\alpha - 2u^2h = 0 \quad ⑤$$

$$gx^2\tan^2\alpha - 2u^2x\tan\alpha + gx^2 - 2u^2h = 0$$

$$\Delta = 4u^4x^2 - 4gx^2(gx^2 - 2u^2h) \quad ⑤$$

$$= 4x^2[u^4 + 2u^2gh - g^2x^2]$$

For real, $\Delta \geq 0$

$$u^4 + 2u^2gh - g^2x^2$$

$$\frac{u^2}{g^2}(u^2 + 2gh) \geq x^2 \quad ⑤$$

$$x_{max} = \frac{u}{g}\sqrt{u^2 + 2gh}$$

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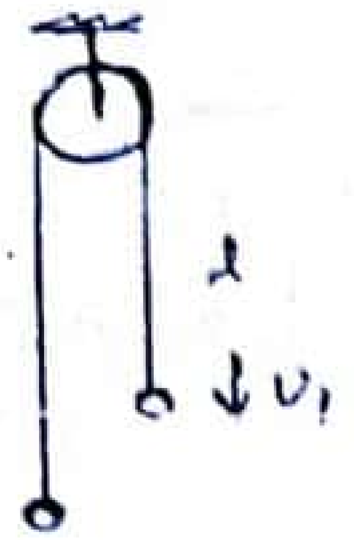
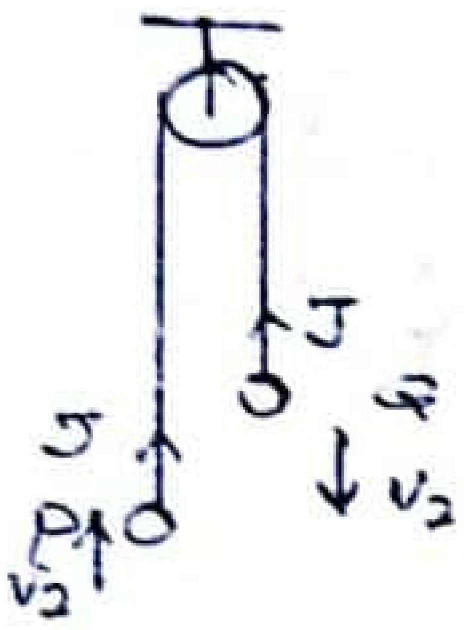
03.





Apply $v^2 = u^2 + 2as \downarrow$ to Q

$v_1^2 = 2gl$

$v_1 = \sqrt{2gl}$ ⑤

Apply $\underline{I} = \Delta(m\underline{v})$ to Q $\downarrow$

$-J = kmv_2 - kmv_1$ — ① ⑤

Apply $\underline{I} = \Delta(m\underline{v})$ to P $\uparrow$

$J = mv_2$ — ② ⑤

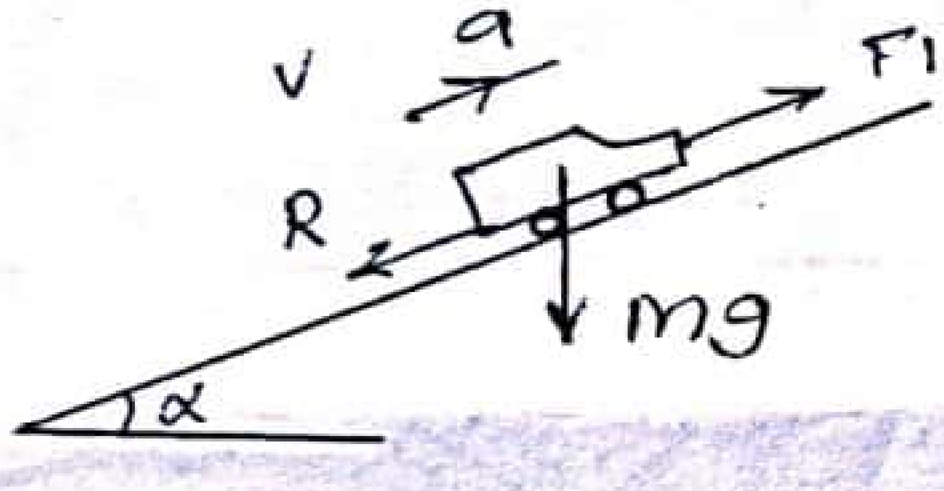
① + ②

$0 = (1+k)v_2 - kv_1$

$v_2 = \frac{k\sqrt{2gl}}{1+k}$ ⑤

$J = \frac{mk\sqrt{2gl}}{1+k}$ ⑤ $\boxed{25}$

04.



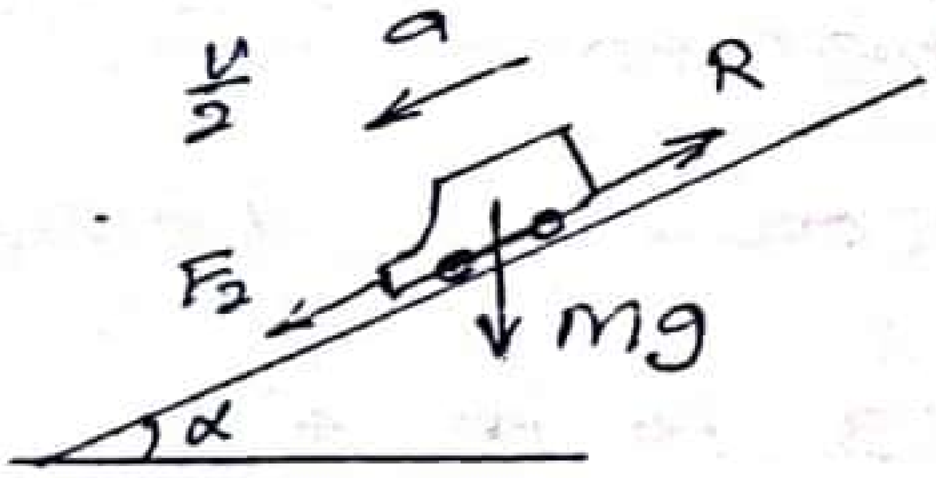
$1000H = F_1 v$

$F_1 = \frac{1000H}{v}$ ⑤

Apply $\underline{F} = m\underline{a}$ $\nearrow$

$F_1 - R - mg\sin\alpha = ma$ ⑤

$\frac{1000H}{v} - R - mg\sin\alpha = ma$ — ①



$1000H = F_2 \cdot \frac{v}{2}$

$F_2 = \frac{2000H}{v}$ ⑤

Apply $\underline{F} = m\underline{a}$ $\swarrow$

$F_2 - R + mg\sin\alpha = ma$ ⑤

$\frac{2000H}{v} - R + mg\sin\alpha = ma$ — ②

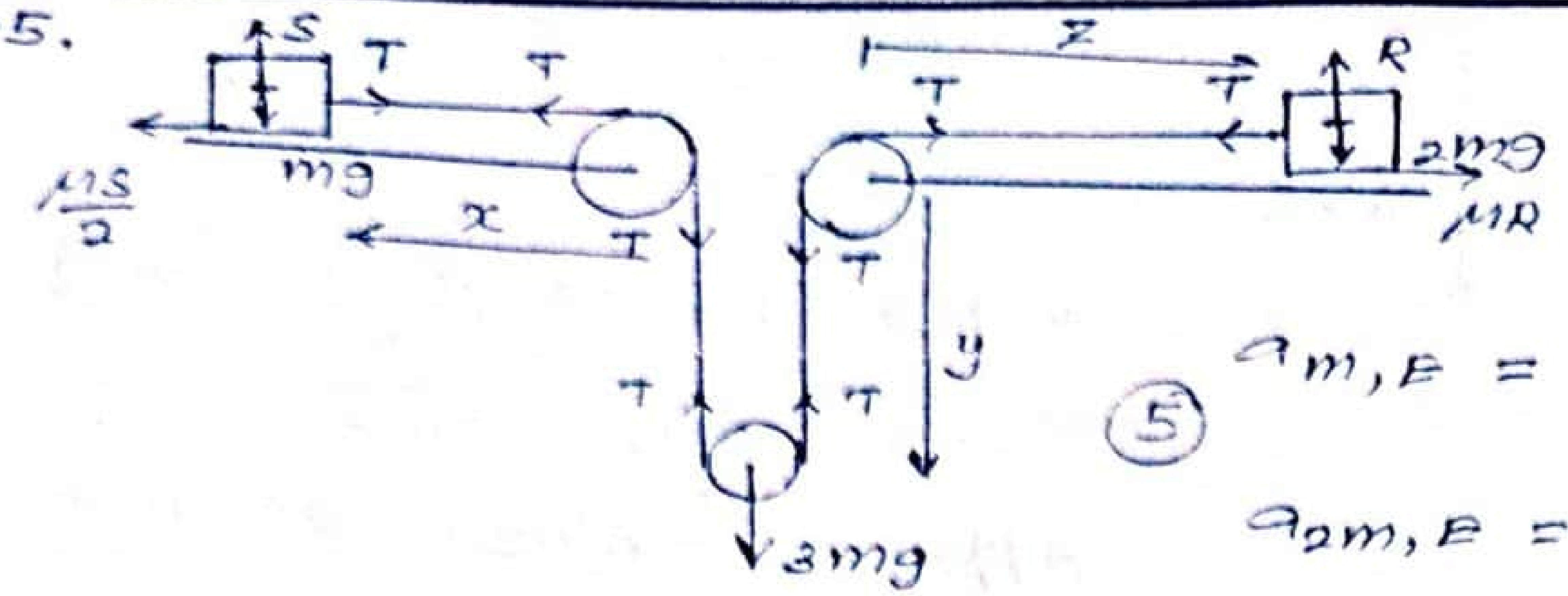
① + ②

$\frac{3000H}{v} - 2R = 2ma$ ⑤

$a = \frac{1500H}{mv} - \frac{R}{m}$

$\boxed{25}$

05.



$a_{m,E} = \overset{\ddot{x}}{\leftarrow}$

$a_{2m,E} = \overset{\ddot{z}}{\rightarrow}$

$a_{3m,E} = \downarrow \ddot{y}$

⑤

Apply F=ma to m ←

$\frac{\mu S}{2} - T = m\ddot{x}$ — ① ⑤

Apply F=ma to 2m →

$R = 2mg$

$S = mg$

$\mu R - T = 2m\ddot{z}$ — ② ⑤

Apply F=ma to 3m ↓

$3mg - 2T = 3m\ddot{y}$ — ③ ⑤

Length of the string

$x + 2\dot{y} + z + l' = l$

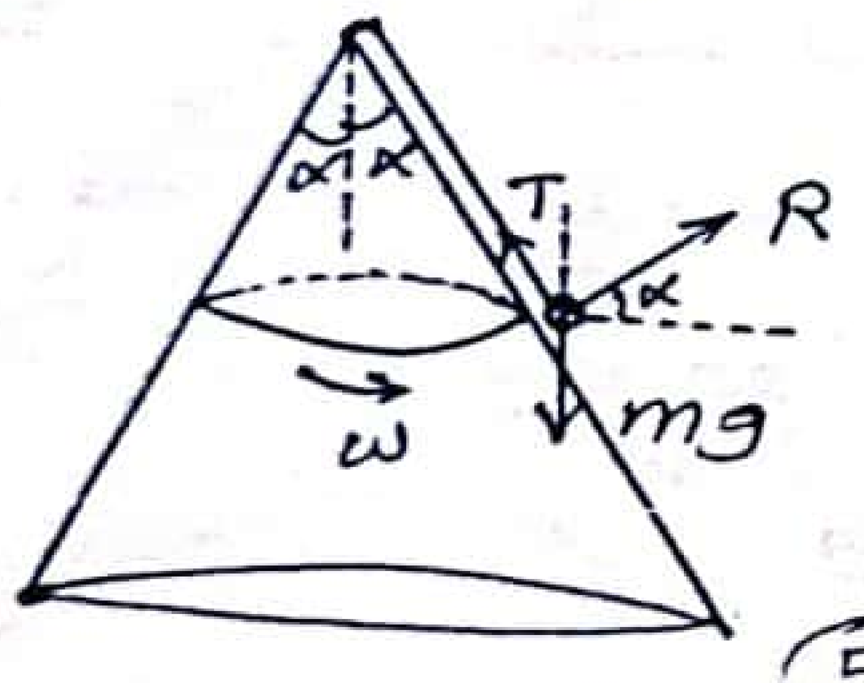
differentiate w.r.t t

$\dot{x} + 2\dot{y} + \dot{z} = 0$

differentiate w.r.t t

$\ddot{x} + 2\ddot{y} + \ddot{z} = 0$ — ④ ⑤

06.



⑤

Apply F=ma to m ← ⑤

$T\sin\alpha - R\cos\alpha = m\,l\sin\alpha\,\omega$

Apply F=ma to m ↑

$T\cos\alpha + R\sin\alpha - mg = 0$ ⑤

$T\cos\alpha + R\sin\alpha = mg$ — ②

① × sinα + ② × cosα

$T = ml\sin^2\alpha\,\omega^2 + mg\cos\alpha$

$= m\left[l\sin^2\alpha\,\omega^2 + g\cos\alpha\right]$ ⑤

② × sinα − ① × cosα

$R = mg\sin\alpha - ml\omega^2\sin\alpha\cos\alpha$

⑤

07. $\underline{a} = 2\underline{i} + \underline{j}$ $\qquad \overrightarrow{AB} = \underline{b} - \underline{a}$

$\underline{b} = -\underline{i} + 2\underline{j}$. $\qquad = -3\underline{i} + \underline{j}$

$\underline{c} = \underline{i} - \underline{j}$ $\qquad \overrightarrow{AC} = \underline{c} - \underline{a}$

$= \underline{i} - 2\underline{j}$ ⑤

New $\qquad$ A, B, C are non-collinear.

$\underline{c} = (\alpha+1)\underline{i} + (\beta-1)\underline{j}$

$\overrightarrow{AC} = (\alpha-1)\underline{i} + (\beta-2)\underline{j}$ ⑤

If A, B, C collinear

$\overrightarrow{AC} = \lambda \overrightarrow{AB}$ ⑤

$(\alpha-1)\underline{i} + (\beta-2)\underline{j} = \lambda[-3\underline{i} + \underline{j}]$ ⑤

$\alpha - 1 = -3\lambda$

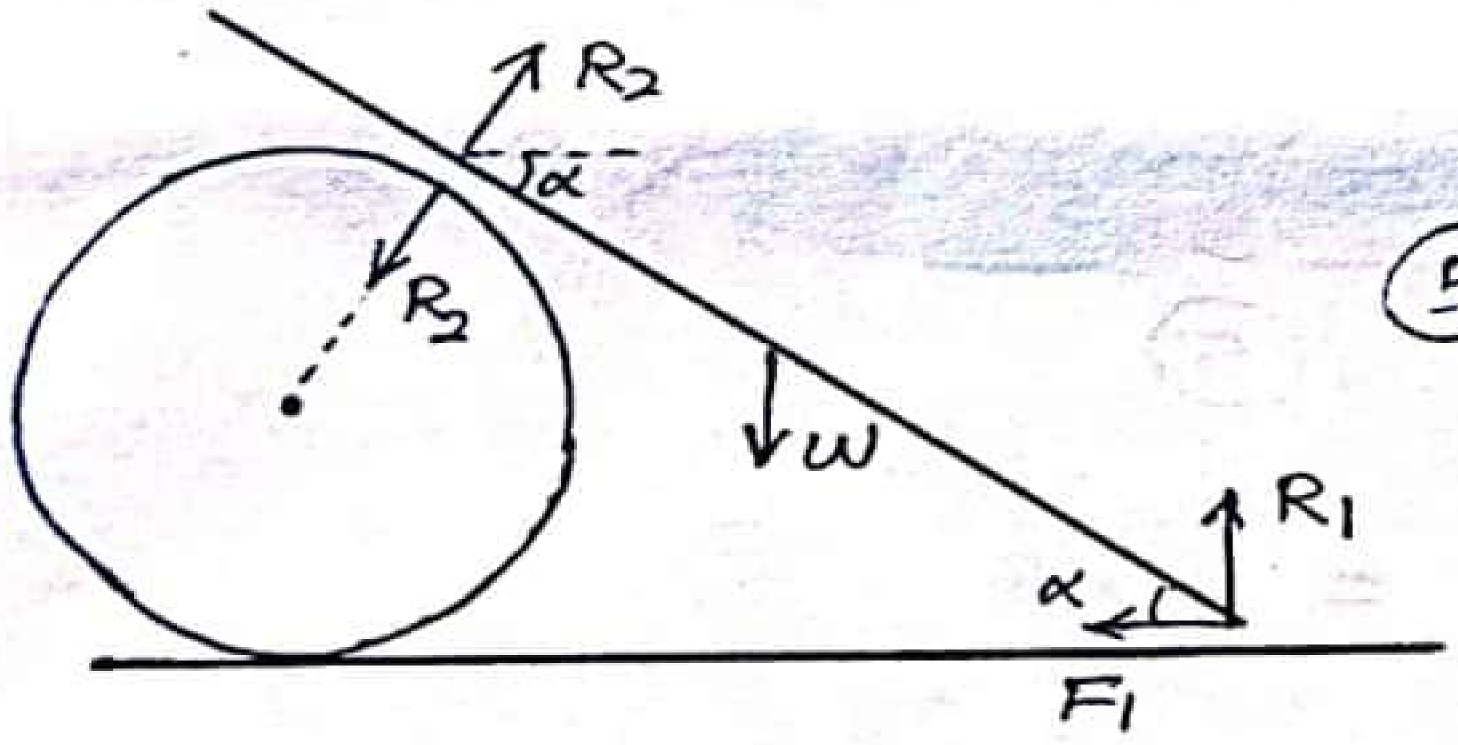
$\beta - 2 = \lambda$

$\alpha - 1 = -3(\beta - 2)$

$\alpha + 3\beta = 7$ ⑤

$\boxed{25}$

08.



⑤

For eqᵐ of rod AB

$\leftarrow \quad F_1 - R_2 \sin\alpha = 0$ ⑤

⑤ $\mu R_1 - R_2 \sin\alpha = 0$

$R_1 = \dfrac{R_2 \sin\alpha}{\mu}$

$\uparrow \quad R_1 + R_2 \cos\alpha - W = 0$ ⑤

$\dfrac{R_2 \sin\alpha}{\mu} + R_2 \cos\alpha = W$

$R_2 = \dfrac{\mu W}{\sin\alpha + \mu\cos\alpha}$ ⑤

$\boxed{25}$

09.

$$P(B'|A') = \frac{P(B' \cap A')}{P(A')}$$

$$= \frac{P(B \cup A)'}{1 - P(A)}$$ ⑤

$$= \frac{1 - P(A \cup B)}{1 - P(A)}$$ ⑤

$$= \frac{1 - [P(A) + P(B) - P(A \cap B)]}{1 - P(A)}$$

$$= 1 - \frac{P(B)}{1 - P(A)}$$ ⑤

$\because P(A \cap B) = 0$

$\because$ A and B mu

exclusive eve

$P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$

$$P(B'|A') = 1 - \frac{1/4}{1 - 1/3}$$

$$= 1 - \frac{3}{8}$$

$$= \frac{5}{8}$$ ⑤

25

10.

$\underline{x_1}$ $\underline{x_2}$ $\underline{4}$ $\underline{5}$ $\underline{5}$

⑤ ⑤

$$\frac{x_1 + x_2 + 4 + 5 + 5}{5} = 3.8$$ ⑤

$$x_1 + x_2 + 14 = 19$$

$$x_1 + x_2 = 5$$

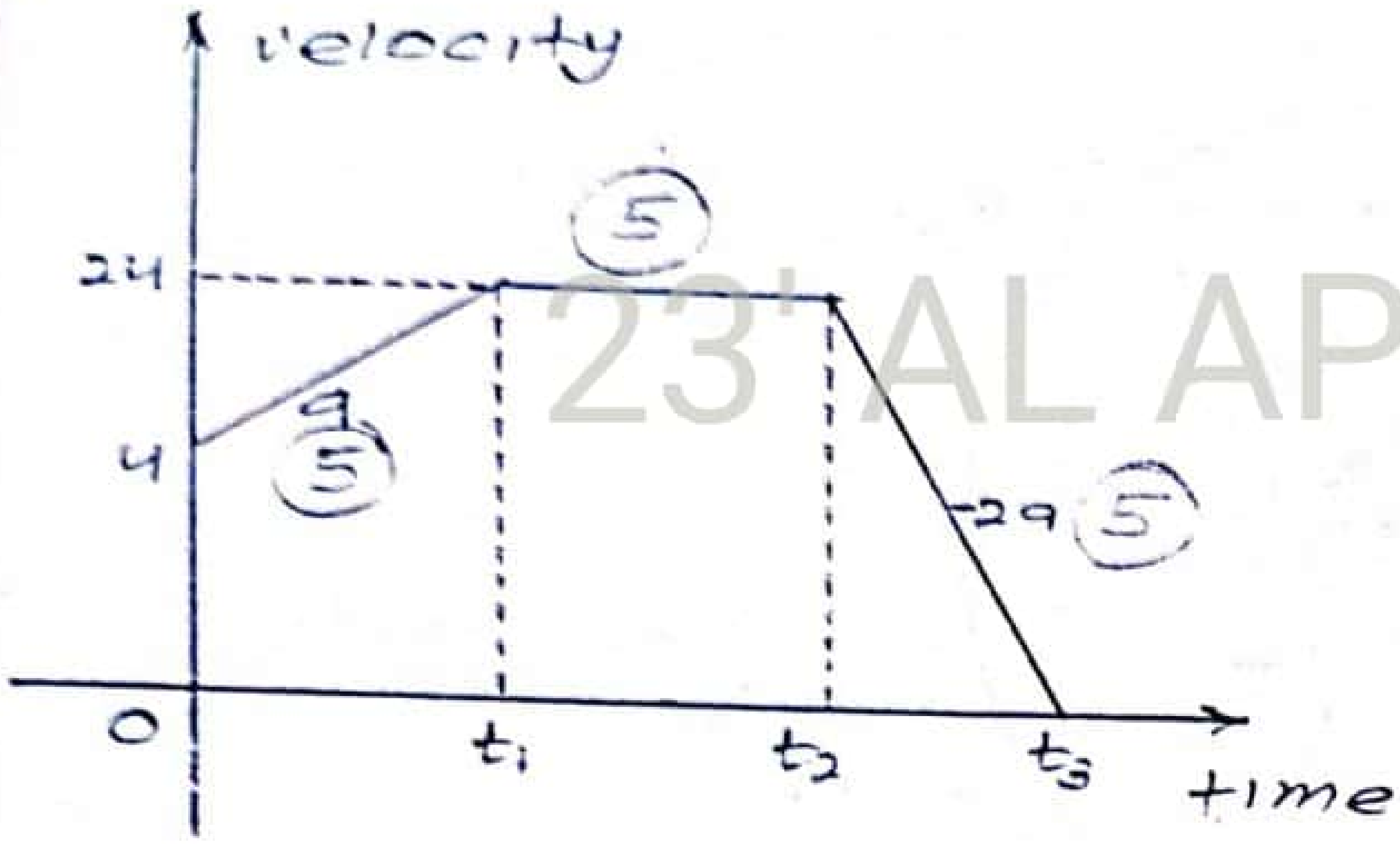
$\Rightarrow 2, 3$ ⑤

observations

2, 3, 4, 5, 5 ⑤

25

11. (a)



$\boxed{15}$

For acc^n

$a = \frac{2u - u}{t_1}$ ⑤ $\Rightarrow \quad t_1 = \frac{u}{a}$ ⑤

For retardation

$-2a = \frac{0 - 2u}{t_3 - t_2}$ ⑤ $\Rightarrow \quad t_3 - t_2 = \frac{u}{a}$ ⑤

For displacement

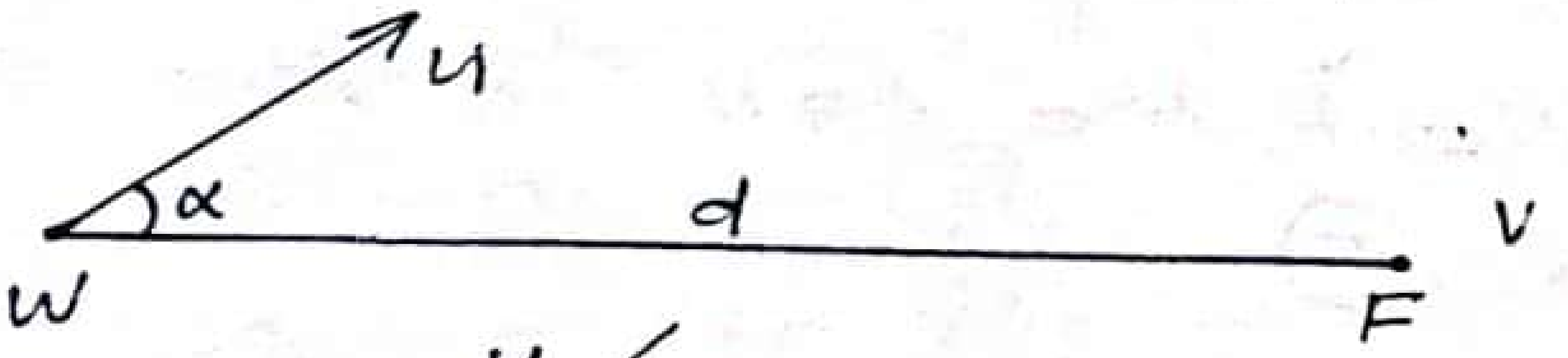
$s = \frac{1}{2}(u + 2u)t_1 + 2u(t_2 - t_1) + \frac{1}{2}(t_3 - t_2)\cdot 2u$ ⑮

$= \frac{1}{2} 3u \cdot \frac{u}{a} + 2u(t_2 - t_1) + \frac{1}{2} \frac{u}{a} \cdot 2u$ ⑤

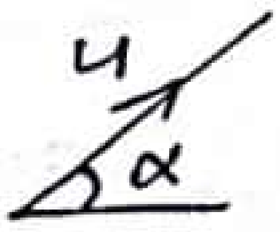
$2u(t_2 - t_1) = s - \frac{3u^2}{2a} - \frac{2u^2}{2a}$

$t_2 - t_1 = \frac{1}{2u}\left[s - \frac{5u^2}{2a} \right]$ ⑤ $\boxed{45}$

b)



$V_{B,E} =$

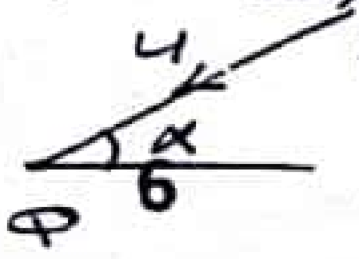


$V_{F,E} = v \quad (u > v)$ ⑤

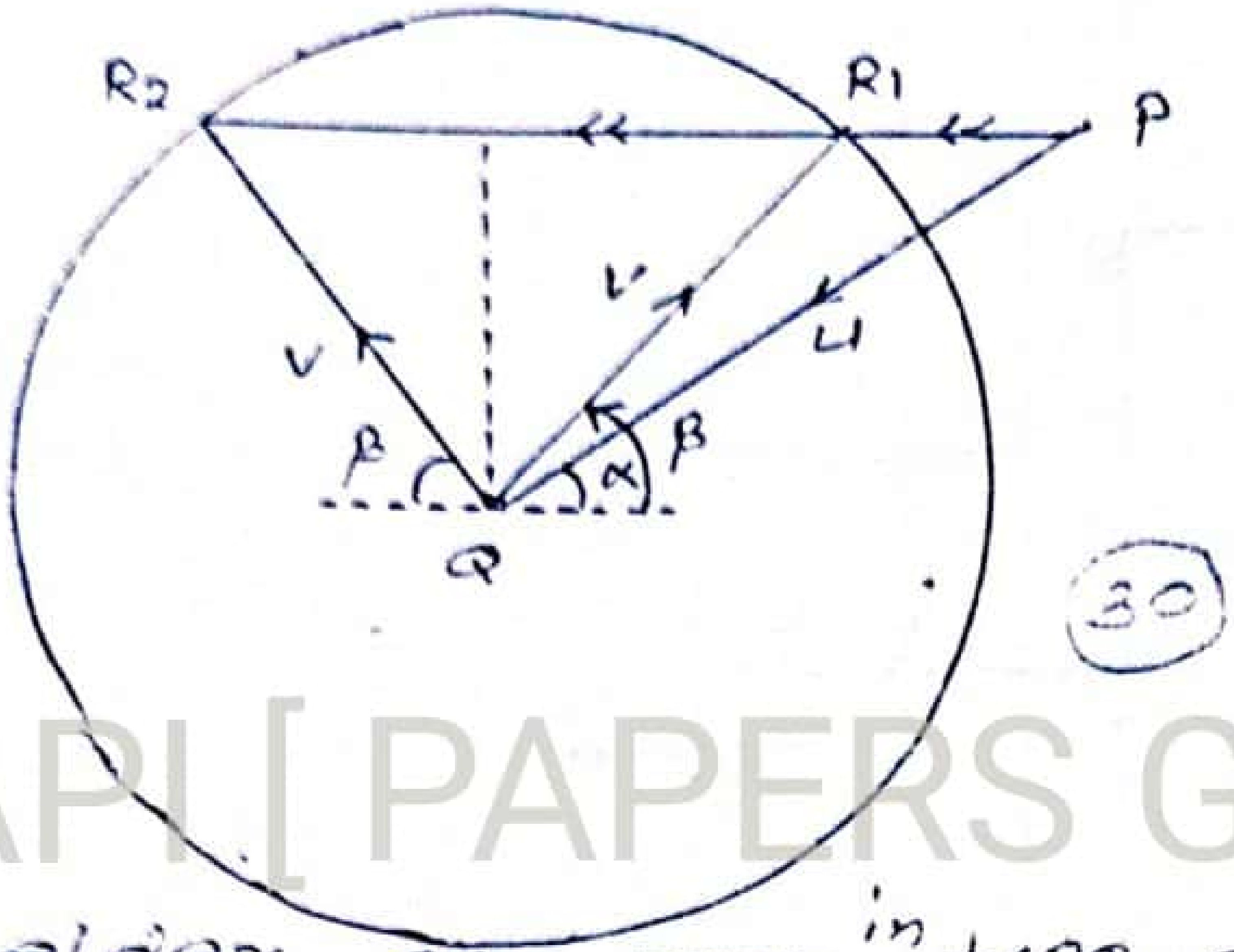
$V_{F,B} = \leftarrow$

$V_{F,B} = V_{F,E} + V_{E,B}$

$\xleftarrow[R \qquad P]{} = v +$



⑤



(30)

4

(i) The fielder can run in two dir^{ns}, QR_1 and QR_2 ⑩

By equating $\perp^r$ distance

$v \sin\beta = u \sin\alpha$

$\beta = \sin^{-1}\left(\frac{u \sin\alpha}{v}\right)$ ⑤

15

(ii) $|\overline{PR_1}| < |\overline{PR_2}|$

$\frac{1}{|\overline{PR_1}|} > \frac{1}{|\overline{PR_2}|}$ ⑤

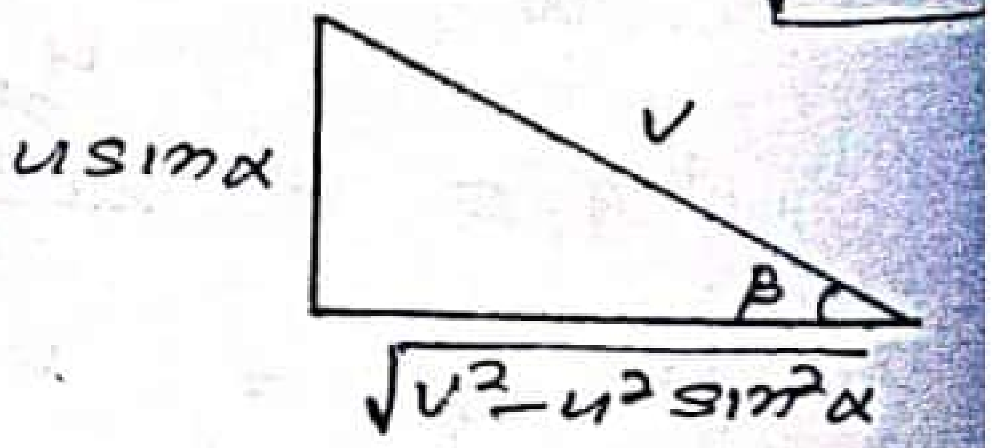
$\frac{d}{|\overline{PR_1}|} > \frac{d}{|\overline{PR_2}|}$

$t_1 > t_2$ ⑤

The fielder should run in the dir^n QR_2 to collect the ball in the shortest time period. ⑤

15

(iii) $t_2 = \frac{d}{u\cos\alpha + v\cos\beta}$ ⑩

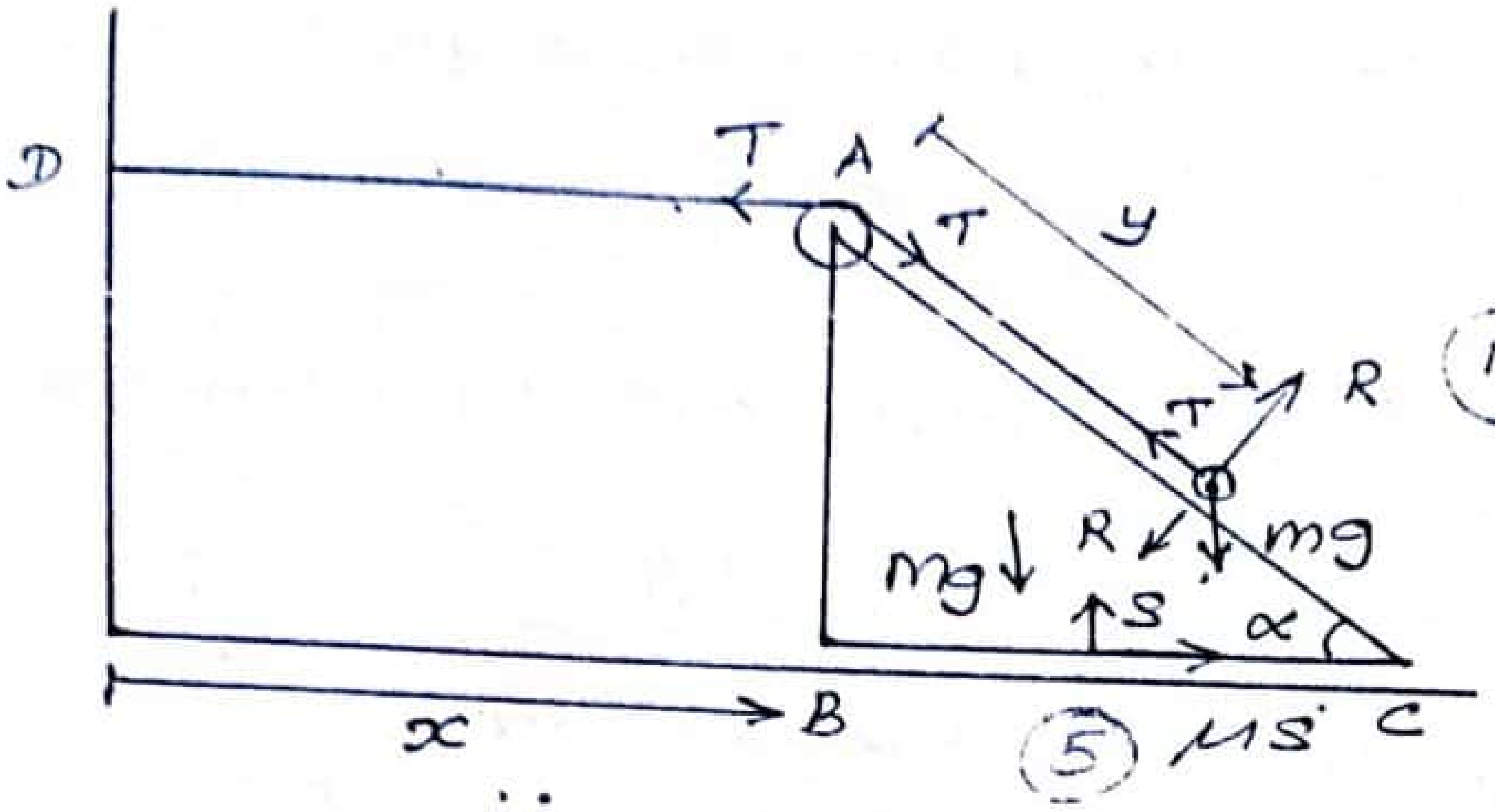


$= \frac{d}{u\cos\alpha + \sqrt{v^2 - u^2\sin^2\alpha}}$ ⑤

$= \frac{d\left[u\cos\alpha - \sqrt{v^2 - u^2\sin^2\alpha}\right]}{u^2 - v^2}$ ⑤

20

12. (a)



⑩ For forces

$a_{m,E} = \xrightarrow{\ddot{x}}$

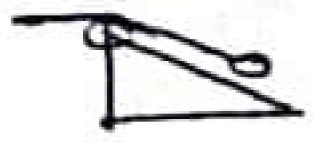
$a_{m,M} =$

⑩ For acc^{ns}

$a_{m,E} = a_{m,M} + a_{M,E}$

$a_{m,E} =$

Apply $\underline{F} = m\underline{a}$ to



$\rightarrow$

$-T + \mu S = M\ddot{x} + m(\ddot{x} + \ddot{y}\cos\alpha)$ — ① ⑩

Apply $\underline{F} = m\underline{a}$ to $m \searrow$

$mg\sin\alpha - T = m(\ddot{y} + \ddot{x}\cos\alpha)$ — ② ⑩

For length of the string

$x + y + l_1 = l$

differentiate w.r.t t

$\dot{x} + \dot{y} = 0$

differentiate w.r.t t

$\ddot{x} + \ddot{y} = 0$ — ③ ⑩

Apply $\underline{F} = m\underline{a}$ to the whole system $\uparrow$

$S - Mg - mg = m(-)\ddot{y}\sin\alpha$ — ④ ⑩

② − ①

$mg\sin\alpha - \mu S = m(\ddot{y} + \ddot{x}\cos\alpha) - M\ddot{x} - m(\ddot{x} + \ddot{y}\cos\alpha)$

$mg\sin\alpha - \mu[(M+m)g - m\ddot{y}\sin\alpha]$

$= m(\ddot{y} + \ddot{x}\cos\alpha) - M\ddot{x} - m(\ddot{x} + \ddot{y}\cos\alpha)$

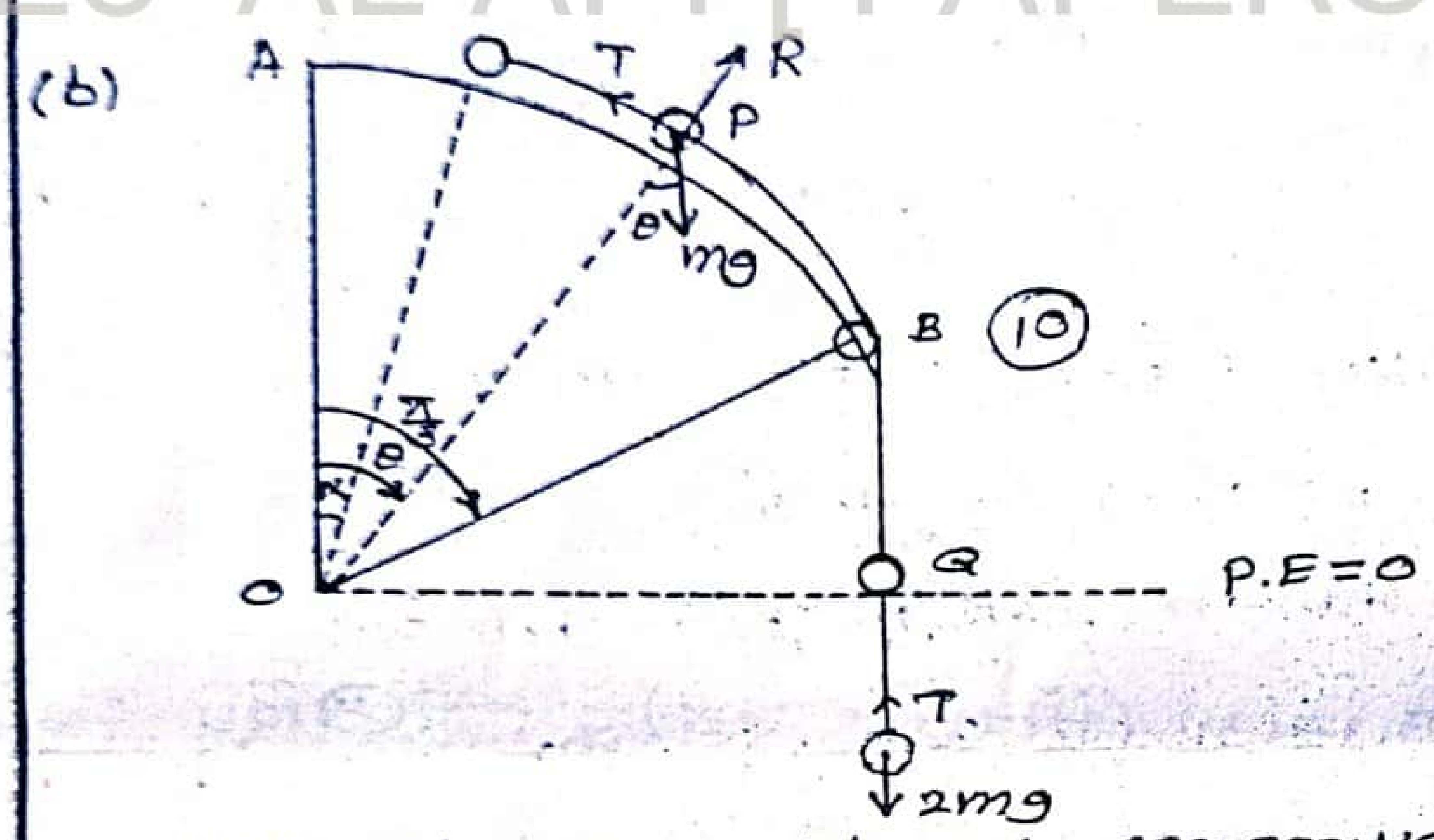
$$mg\sin\alpha-(M+m)\mu g = m\mu\sin\alpha\,\ddot{x} + m(-\ddot{x}+\ddot{x}\cos$$

$$-M\ddot{x}-m\ddot{x}+m\ddot{x}\cos\alpha$$

$$= \ddot{x}[m\mu\sin\alpha - M - 2m + 2m\cos$$

$$\ddot{x} = \frac{[m\sin\alpha-(M+m)\mu]g}{m(\mu\sin\alpha+2\cos\alpha-2)-M}$$ ⑩ $\boxed{7}$

(b)



Apply the principle of conservation of energ

$$0 + mga\cos\alpha = \frac{1}{2}mv^2 + \frac{1}{2}2mv^2 + mga\cos\theta - 2mga($$ ⑤ ⑤ ⑤ ⑤ ⑤

$$ga\cos\alpha = \frac{3v^2}{2} + ga\cos\theta - 2ga(\theta-\alpha)$$

$$\frac{3}{2}a^2\dot{\theta}^2 = \frac{2ga}{3}[\cos\alpha - \cos\theta + 2\theta - 2\alpha]$$ ⑤

$$a\dot{\theta}^2 = \frac{2g}{3}[\cos\alpha - \cos\theta + 2\theta - 2\alpha]$$ ⑤

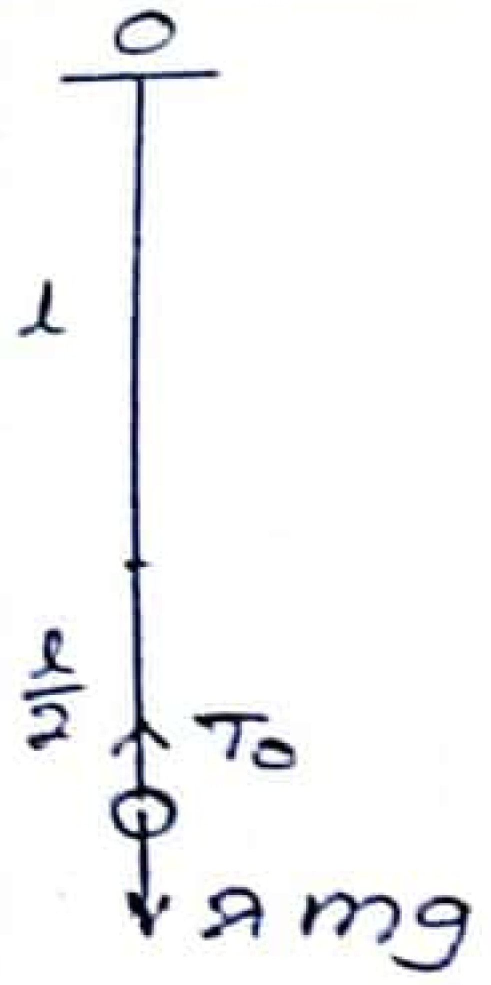
Apply $\underline{F}=m\underline{a}$ to m ↙

$$mg\cos\theta - R = m\frac{v^2}{a}$$ ⑩ ⑤

$$R = mg\cos\theta - \frac{m}{a}\frac{2ga}{3}[\cos\alpha - \cos\theta + 2\theta - 2\alpha$$

$$= \frac{2mg}{3}\left[\frac{5}{2}\cos\theta - \cos\alpha - 2\theta + 2\alpha\right]$$ ⑩

$\boxed{75}$

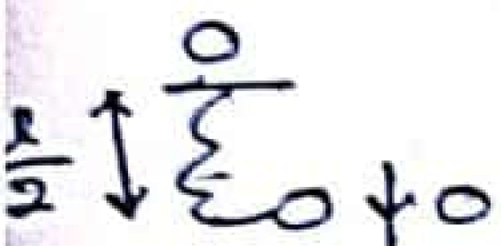


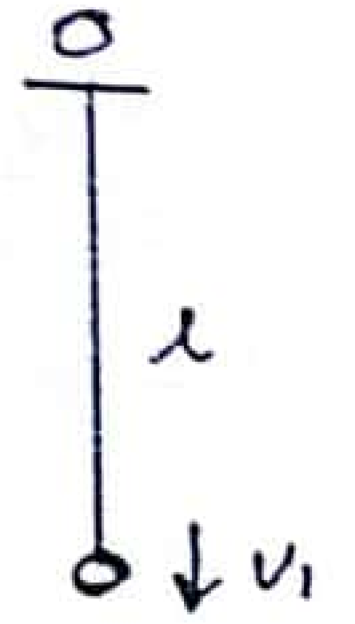
For eq^m

$T_0 = \lambda mg$ ⑤

⑤ $k \cdot \frac{l/2}{l} = \lambda mg$

$k = 2\lambda mg$ ⑤ $\boxed{15}$

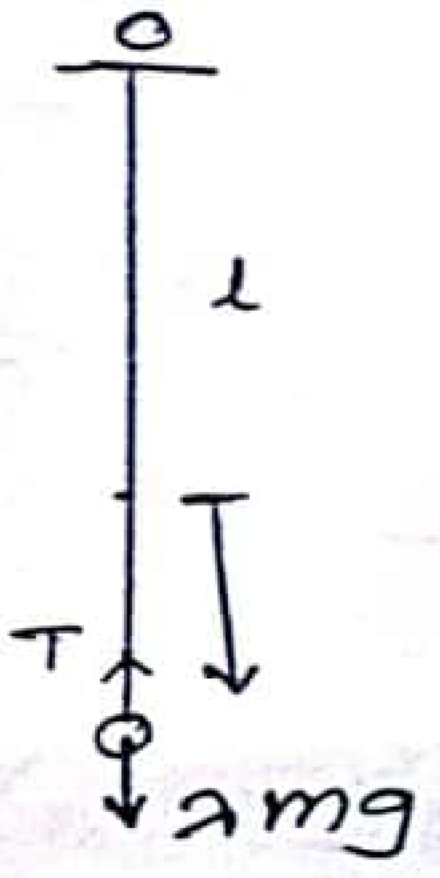




Apply $v^2 = u^2 + 2as$ to $\lambda m \downarrow$

$v_1^2 = 0 + 2g\frac{l}{2}$ ⑤

$v_1 = \sqrt{gl}$ ⑤ $\boxed{10}$



Apply $\underline{F} = m\underline{a}$ to $\lambda m \downarrow$

$\lambda mg - T = \lambda m \ddot{x}$ ⑩

⑤
$\lambda mg - 2\lambda mg \frac{x}{l} = \lambda m \ddot{x}$

$\ddot{x} + \frac{2g}{l}\left(x - \frac{l}{2}\right) = 0$ ⑤ $\boxed{20}$

For centre $\ddot{x} = 0$,

$-\frac{2g}{l}\left(x - \frac{l}{2}\right) = 0$

$x = \frac{l}{2}$ ⑤

The centre lies distance $\frac{3l}{2}$ below O

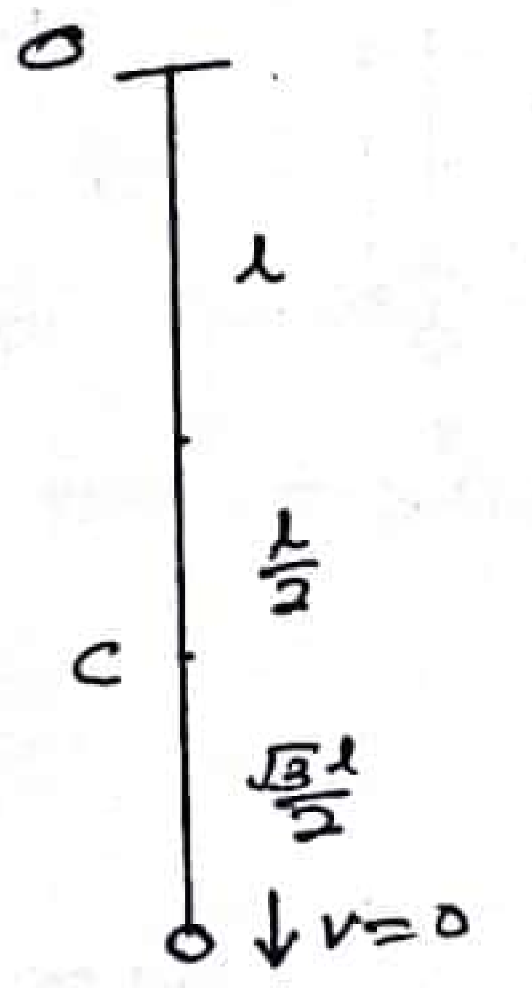
$\dot{x}^2 = \omega^2(A^2 - x^2)$

when $x = -\frac{l}{2}$, $\dot{x} = \sqrt{gl}$ ⑤

$gl = \frac{2g}{l}\left(A^2 - \frac{l^2}{4}\right)$

$A = \frac{\sqrt{3}\,l}{2}$ ⑤

Amplitude $A = \frac{\sqrt{3}\,l}{2}$



max^m distance

$= \frac{l}{2}(3+\sqrt{3})$ ⑤ $\boxed{20}$

For the free motion of the particle und
gravity

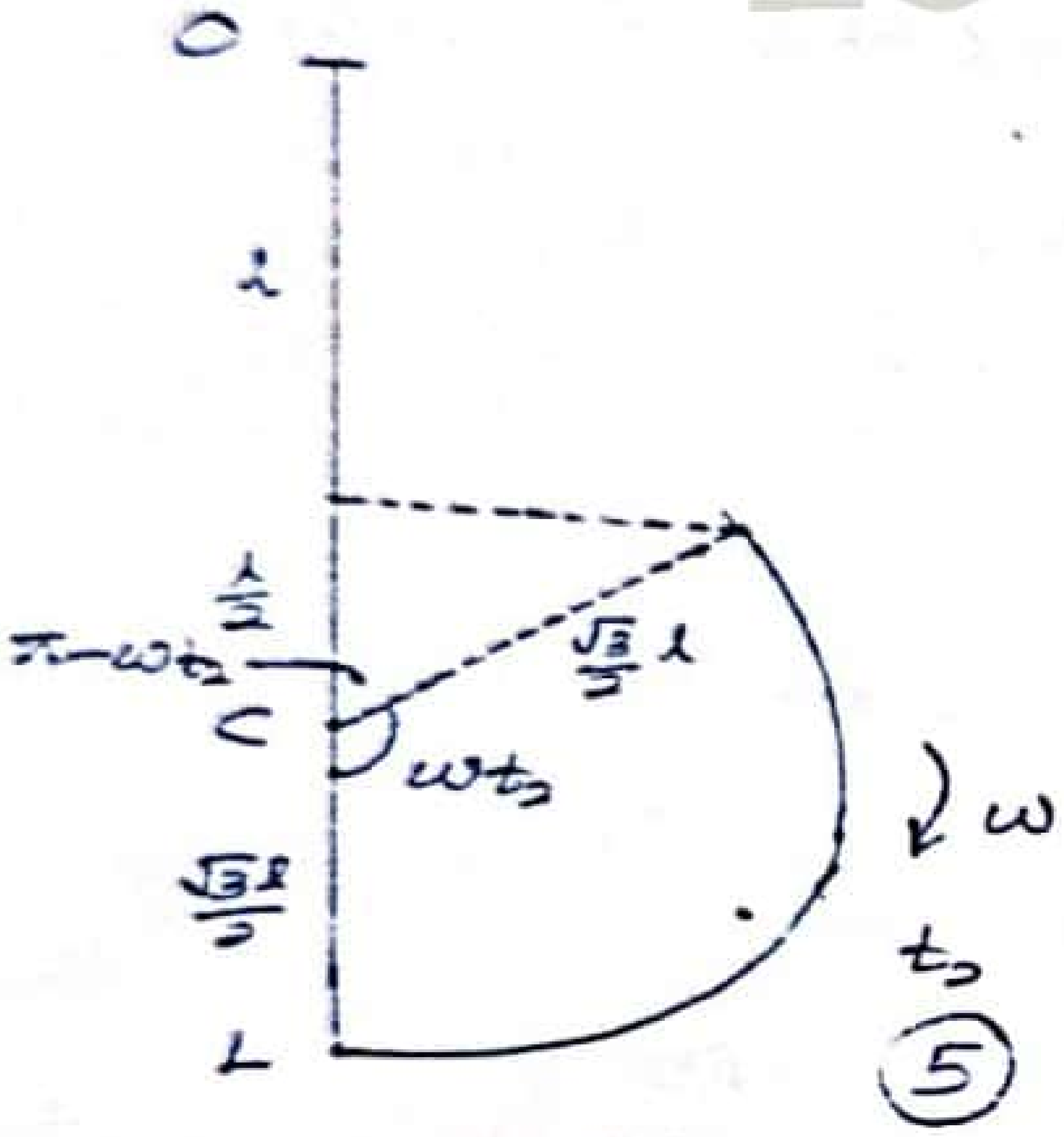
Apply $v = u + at \downarrow$

$\sqrt{gl} = 0 + gt_1$ ⑤

$t_1 = \sqrt{\frac{l}{g}}$ ⑤

$\boxed{1}$

For SHM.



$\cos(\pi - \omega t_2) = \frac{l/2}{\sqrt{3}l/2}$ ⑤

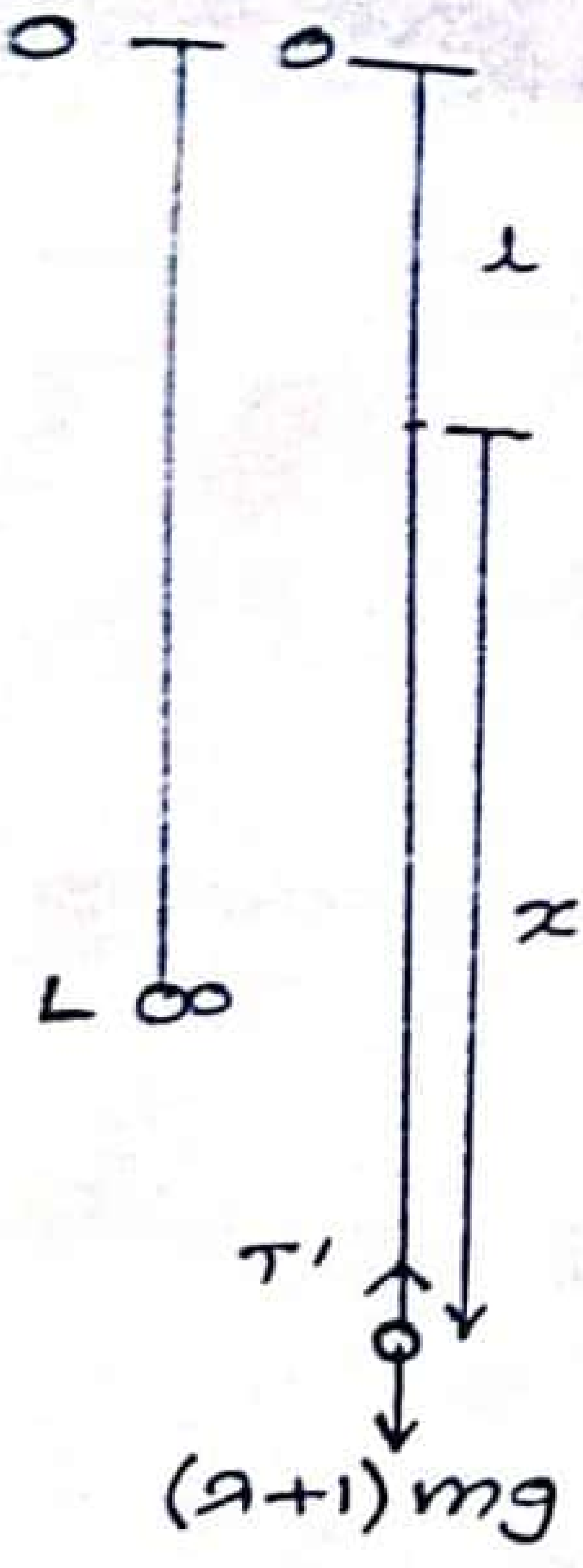
$\pi - \omega t_2 = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ ⑤

$t_2 = \sqrt{\frac{l}{2g}}\left[\pi - \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)\right]$

Total time $= t_1 + t_2$ ⑤

$= \sqrt{\frac{l}{g}}\left\{1 + \frac{1}{\sqrt{2}}\left[\pi - \cos^{-1}\left(\frac{1}{\sqrt{3}}\right.\right.\right.$

$\boxed{2}$



Apply $\underline{F} = m\underline{a}$ to $(\lambda+1)m \downarrow$

$(\lambda+1)mg - T' = (\lambda+1)m\ddot{x}$ ⑩

⑤

$(\lambda+1)mg - 2\lambda mg\frac{x}{l} = (\lambda+1)m\ddot{x}$

$-\frac{2\lambda g}{l}\left[x - \frac{(\lambda+1)l}{2\lambda}\right] = (\lambda+1)\ddot{x}$

⑤

$\ddot{x} + \frac{2\lambda g}{(\lambda+1)l}\left[x - \frac{(\lambda+1)l}{2\lambda}\right] = 0$

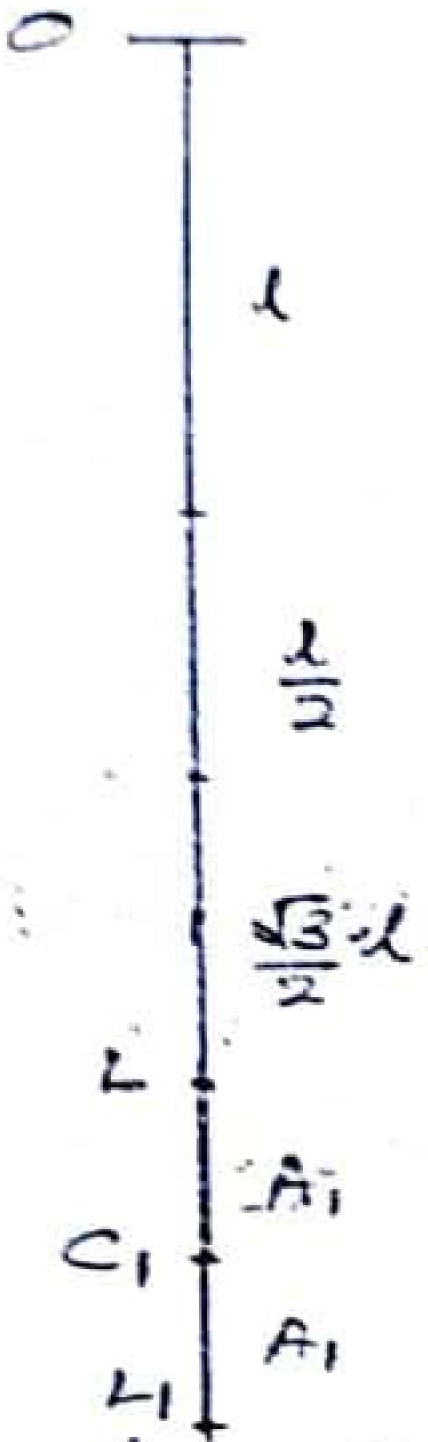
For centre $\ddot{x} = 0$

$\frac{-2\lambda g}{(\lambda+1)l}\left[x - \frac{(\lambda+1)l}{2\lambda}\right] = 0$

$\Rightarrow x = \frac{(\lambda+1)l}{2\lambda}$ ⑤

The distance from O to the new cent

$= l + \frac{(\lambda+1)l}{2\lambda} = \frac{l}{2\lambda}(3\lambda+1)$ ⑤ $\boxed{30}$



Amplitude of new motion A_1

$$\dot{x}^2 = \omega^2(A_1^2 - x^2)$$

$$0 = \omega^2\left[A_1^2 - \left(\frac{\sqrt{3}l}{2} - \frac{l}{2a}\right)^2\right]$$ ⑤

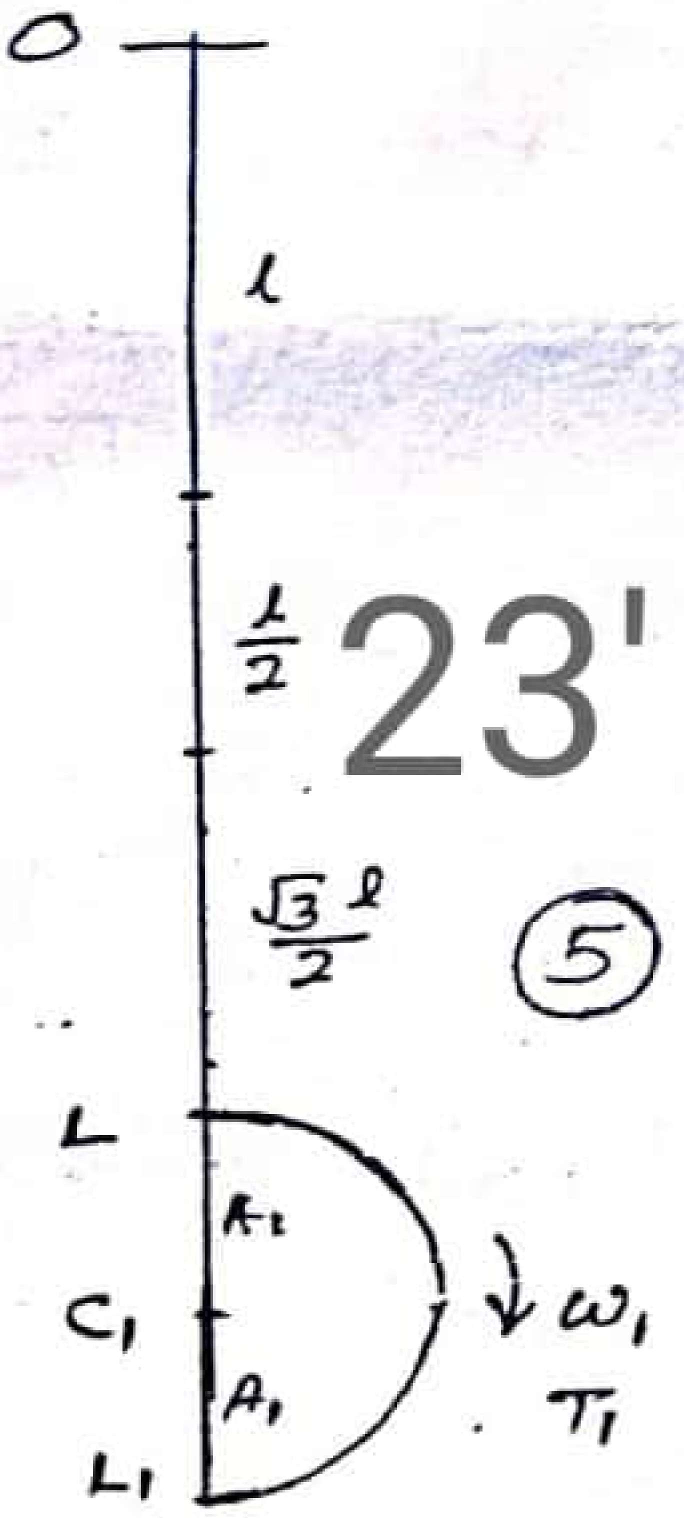
$$A_1 = \frac{\sqrt{3}l}{2} - \frac{l}{2a}$$

$$= \frac{l(\sqrt{3}a - 1)}{2a}$$ ⑤

Let T_1 be the time taken to the new motion of the particle from L to the lowest point L_1.

$$T_1 = \frac{\pi}{\omega_1}$$

$$= \pi\sqrt{\frac{(a+1)l}{2ag}}$$ ⑤



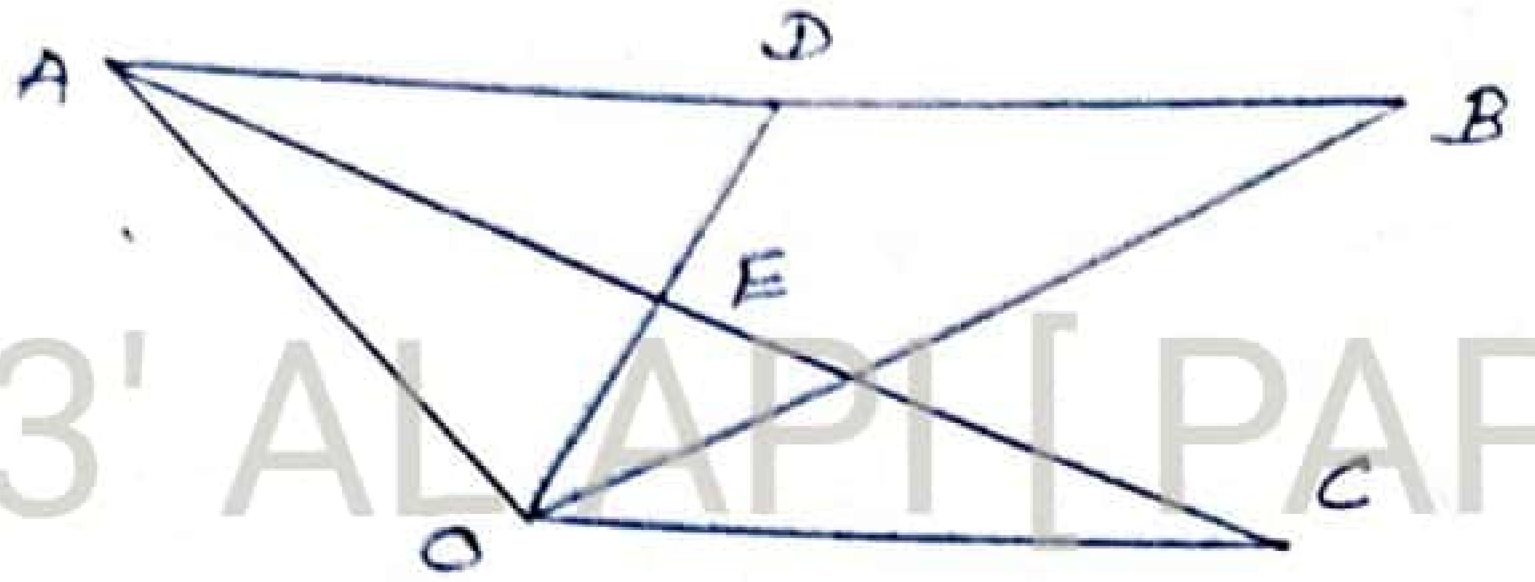
The maximum distance travelled by the composite particle from O is

$$l + \frac{l}{2} + \frac{\sqrt{3}l}{2} + 2\left(\frac{\sqrt{3}}{2}l - \frac{l}{2a}\right)$$

$$= \frac{l}{2a}\left[3(1+\sqrt{3})a - 2\right]$$ ⑤

$\boxed{25}$

14. (a)



$\overrightarrow{OA} = \underline{a}$

$\overrightarrow{OB} = \underline{b}$

Apply triangle law to Δ OAD,

$$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD}$$

$$= \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB}$$

$$= \overrightarrow{OA} + \frac{1}{2}(\overrightarrow{AO} + \overrightarrow{OB}) \quad ⑤$$

$$= \underline{a} - \frac{1}{2}\underline{a} + \frac{1}{2}\underline{b}$$

$$= \frac{\underline{a} + \underline{b}}{2} \quad ⑤$$

1

$$\overrightarrow{OC} = k\,\overrightarrow{AB}$$

$$= k(\overrightarrow{AO} + \overrightarrow{OB})$$

$$= k(-\underline{a} + \underline{b})$$

$$= k(\underline{b} - \underline{a}) \quad ⑤$$

$$\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC} \quad ⑤.$$

$$= -\underline{a} + k(\underline{b} - \underline{a}$$

$$= k\underline{b} - (1+k)\underline{a}$$

1

For Δ OAE

$$\overrightarrow{OE} = \overrightarrow{OA} + \overrightarrow{AE}$$

$$\lambda\overrightarrow{OD} = \overrightarrow{OA} + \mu\overrightarrow{AC} \quad ⑤$$

$$\frac{\lambda}{2}(\underline{a} + \underline{b}) = \underline{a} + \mu[k\underline{b} - (1+k)\underline{a}] \quad ⑤$$

$$\frac{\lambda}{2} = 1 - \mu - k\mu \quad — ①$$

$$\frac{\lambda}{2} = \mu k \quad \Rightarrow \quad \mu = \frac{\lambda}{2k}$$

$$① \Rightarrow \quad \frac{\lambda}{2} = 1 - \frac{\lambda}{2k} - k \cdot \frac{\lambda}{2k}$$

$$\lambda + \frac{\lambda}{2k} = 1 \quad ⑤$$

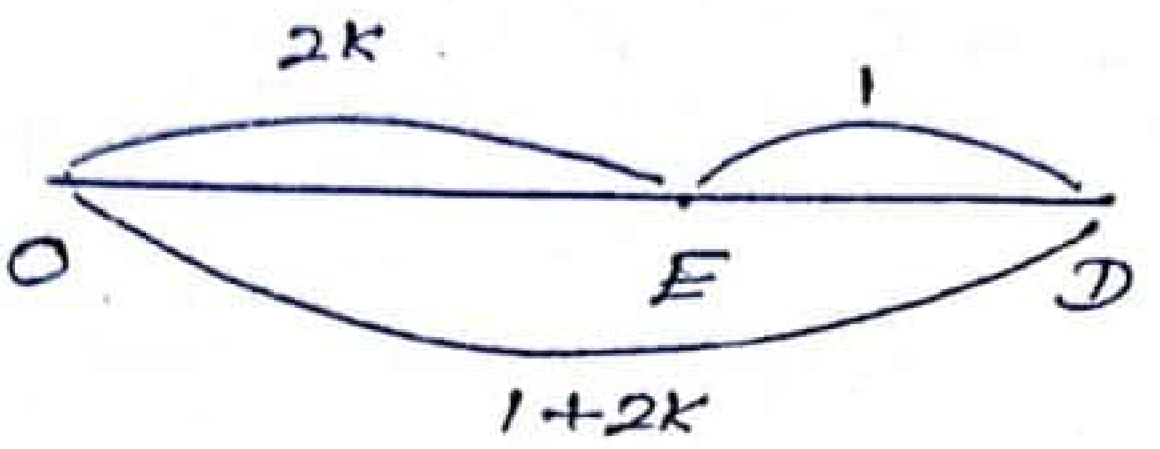
$$\lambda = \frac{2k}{1+2k}$$

$$\mu = \frac{1}{1+2k}$$

$$\overrightarrow{OE} = \frac{2k}{1+2k}\overrightarrow{OD}$$

$$|\overrightarrow{OE}| = \frac{2k}{1+2k}|\overrightarrow{OD}|$$ ⑤

$$\frac{OE}{OD} = \frac{2k}{1+2k}$$

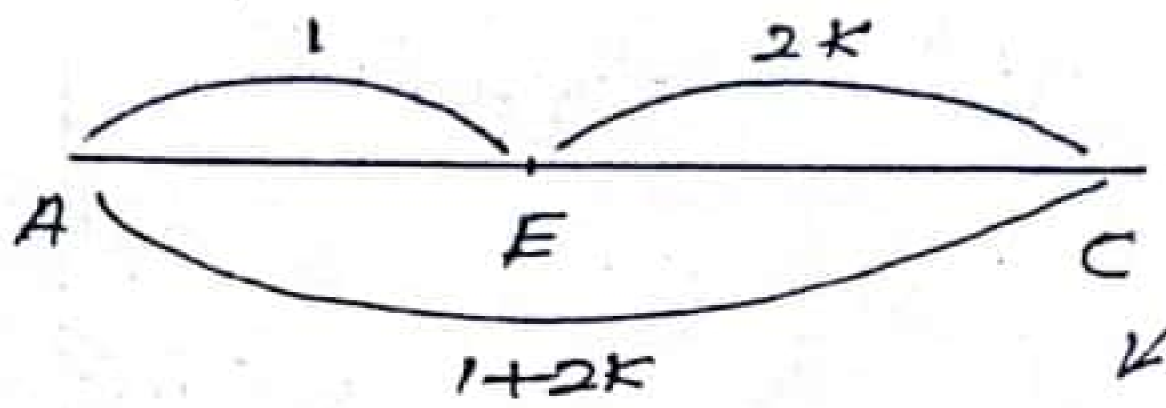


OE : ED = 2k : 1

$$\overrightarrow{AE} = \frac{1}{1+2k}\overrightarrow{AC}$$

$$|\overrightarrow{AE}| = \frac{1}{1+2k}|\overrightarrow{AC}|$$

$$\frac{AE}{AC} = \frac{1}{1+2k}$$ ⑤



⑤ $\boxed{30}$

AE : EC = 1 : 2k

since $\overrightarrow{OD} \perp^r \overrightarrow{AC}$

$\overrightarrow{OD}\cdot\overrightarrow{AC} = 0$ ⑤

$$\frac{\underline{a}+\underline{b}}{2}\cdot[k\underline{b} - (1+k)\underline{a}] = 0$$

$$k(\underline{a}\cdot\underline{b}) + k(\underline{b}\cdot\underline{b}) - (1+k)(\underline{a}\cdot\underline{a}) - (1+k)(\underline{a}\cdot\underline{b}) = 0$$ ⑤

$$\underline{a}\cdot\underline{b} = k|\underline{b}|^2 - (1+k)|\underline{a}|^2$$

$$= k[|\underline{b}|^2 - |\underline{a}|^2] - |\underline{a}|^2$$

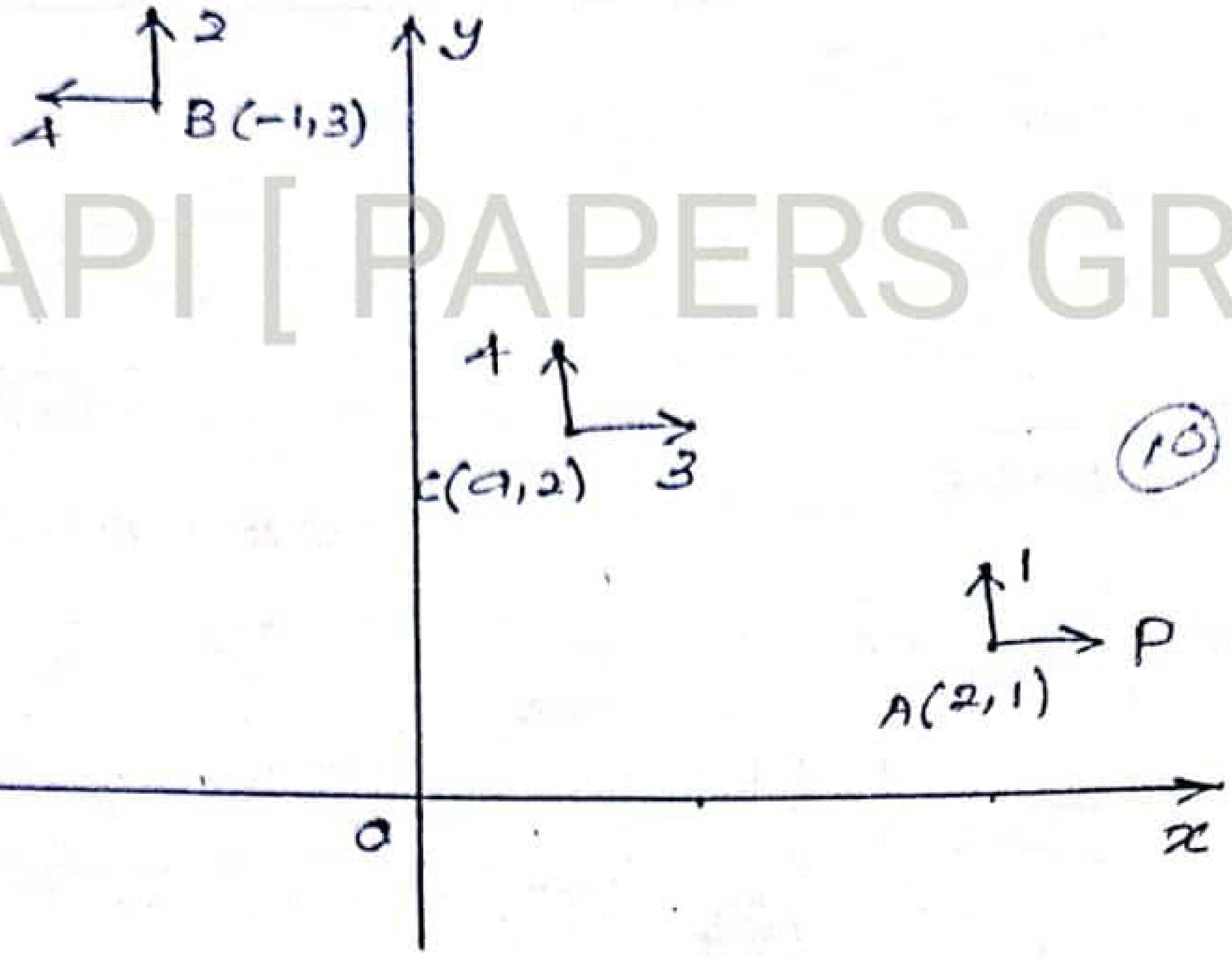
since $\underline{a}$ and $\underline{b}$ are perpendicular to each other

$\underline{a}\cdot\underline{b} = 0$ ⑤

$$k[|\underline{b}|^2 - |\underline{a}|^2] - |\underline{a}|^2 = 0$$

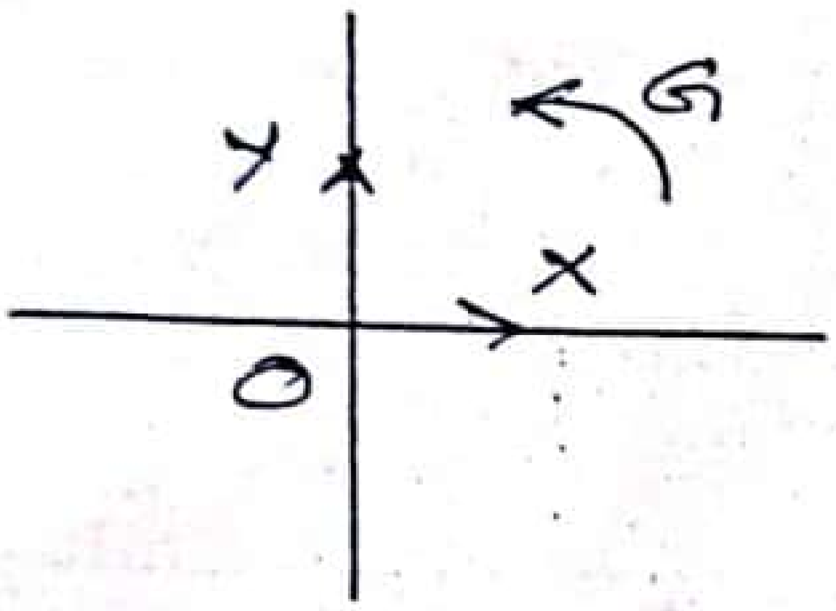
$$k = \frac{|\underline{a}|^2}{|\underline{b}|^2 - |\underline{a}|^2}$$ ⑤ $\boxed{20}$

(b) (i)



⑩

≡



$\boxed{10}$

(ii) $\rightarrow$ $X = P + 3 - 4 = P - 1$ ⑤

$\uparrow$ $Y = 2 + 1 + 4 = 7$ ⑤

$O \circlearrowleft$ $G = -P.1 + 1.2 - 3.2 + 4.a + 4.3 - 2.1$

$= 4a - P + 6$ ⑤

$\boxed{15}$

(iii) $\tan 45 = \frac{Y}{X}$ ⑤

$1 = \frac{7}{P-1}$ $\Rightarrow$ $P = 8$ ⑤

reduced only to a single force

$R \neq 0$, $G = 0$

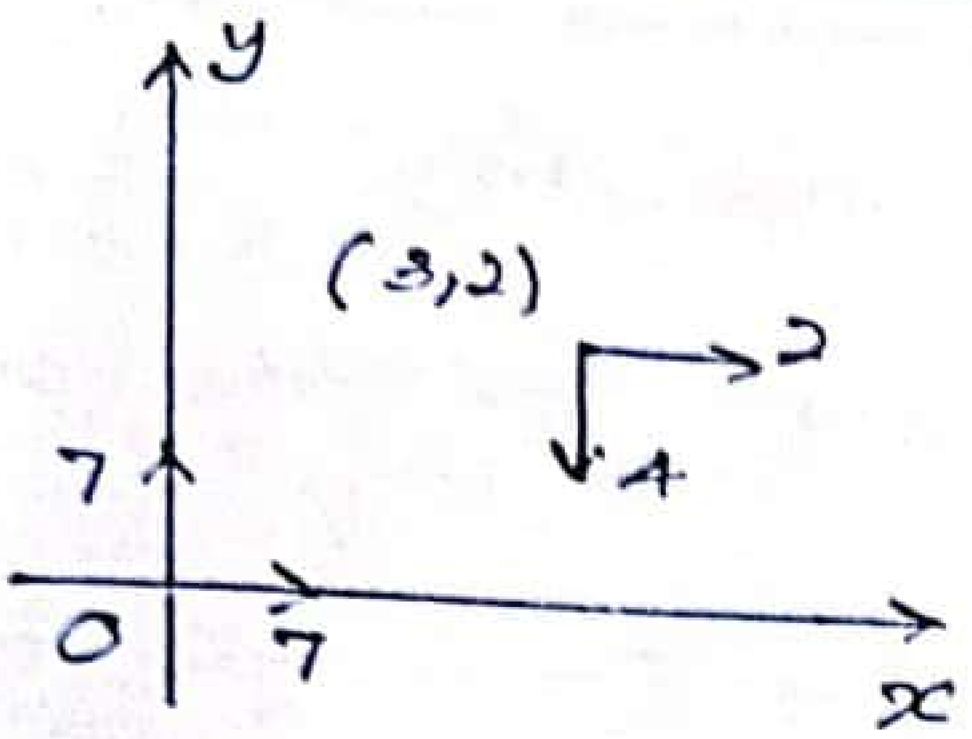
$4a - P + 6 = 0$

$4a - 8 + 6 = 0$

$a = \frac{1}{2}$ ⑤

$\boxed{15}$

iv)



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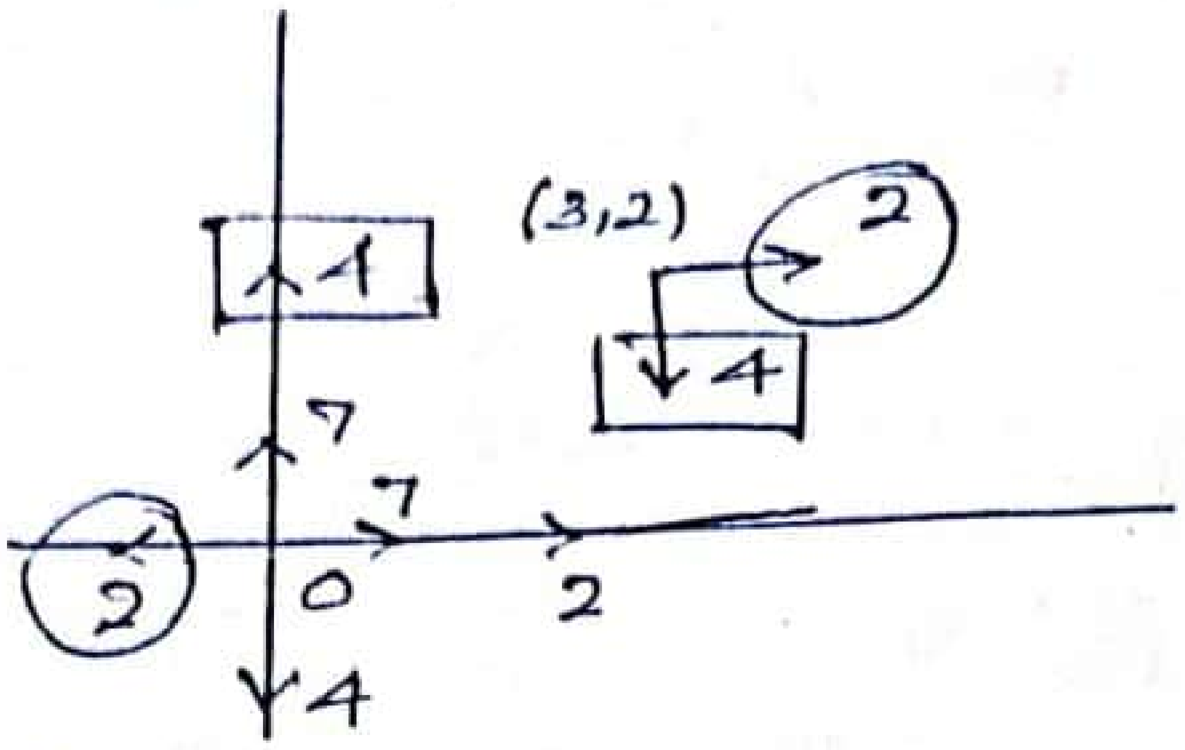
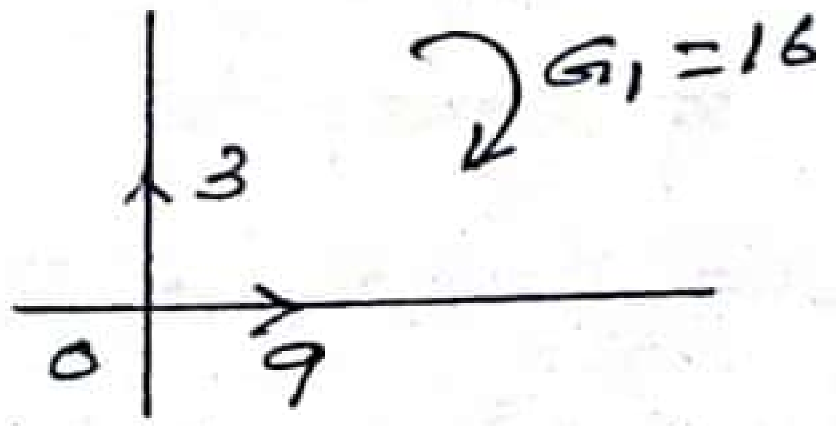


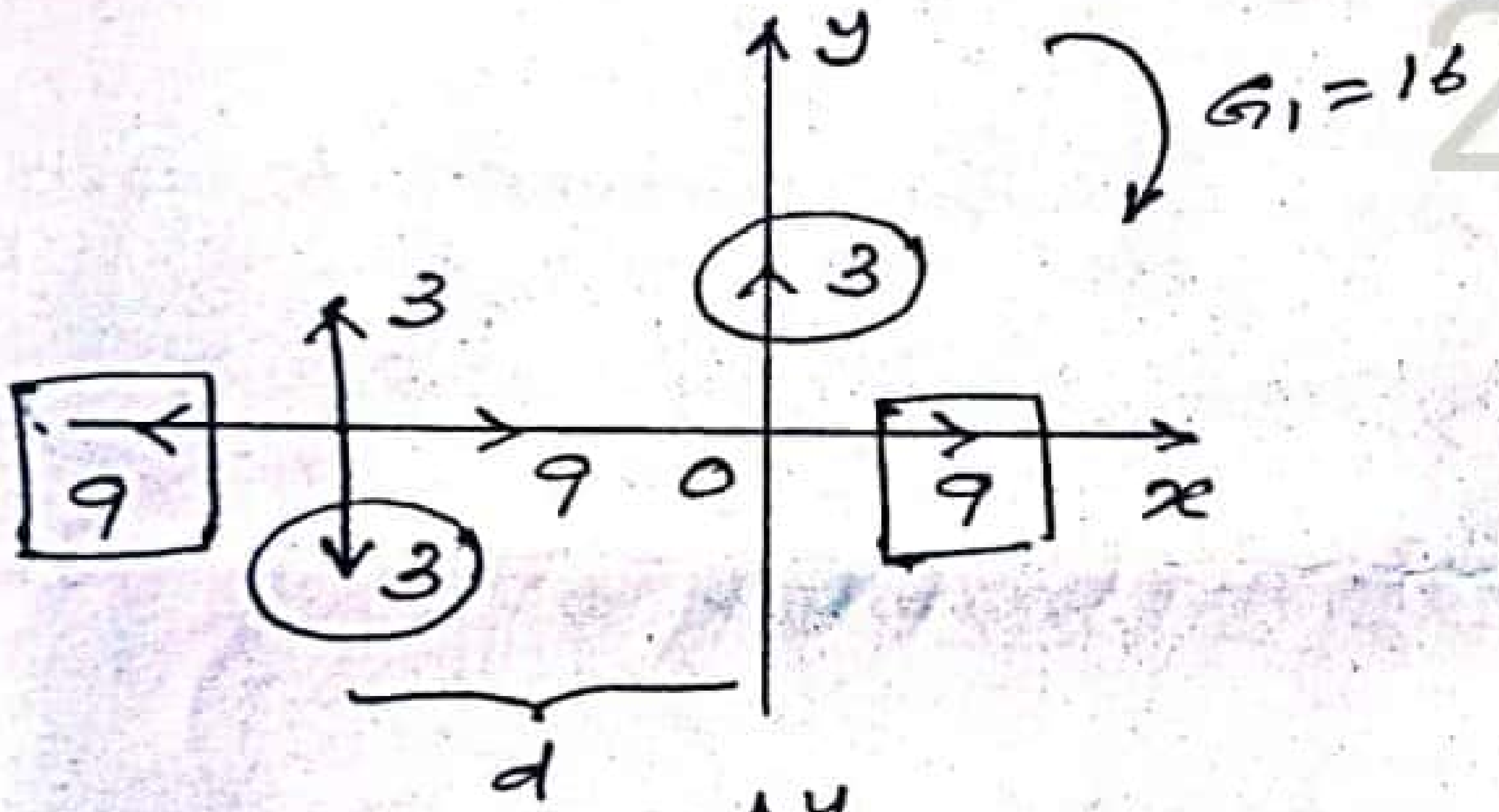
$\circlearrowright G_1 = 4\times3 + 2\times2$

$= 16$ ⑤

≡



≡



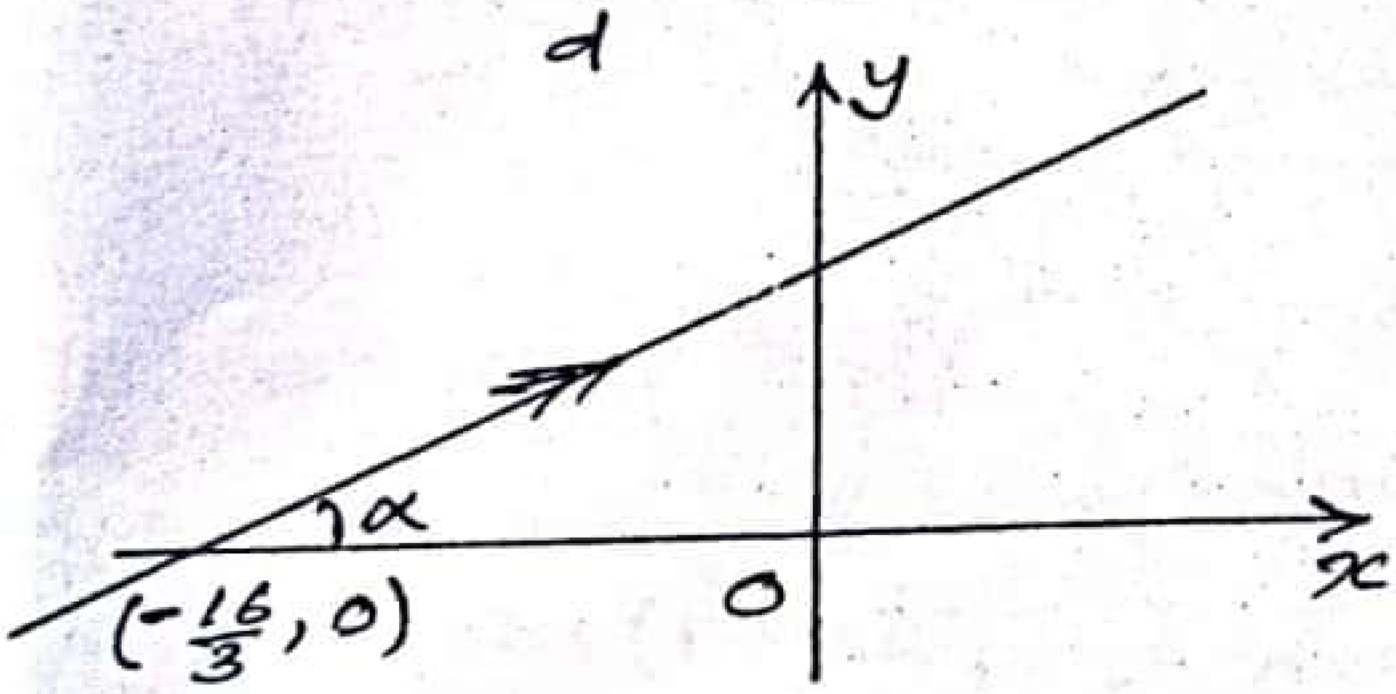
$3\times d = 16$

$d = \frac{16}{3}$ ⑤

$\tan\alpha = \frac{3}{9} = \frac{1}{3}$ ⑤

$R = \sqrt{9^2+3^2} = \sqrt{90}$ ⑤

≡

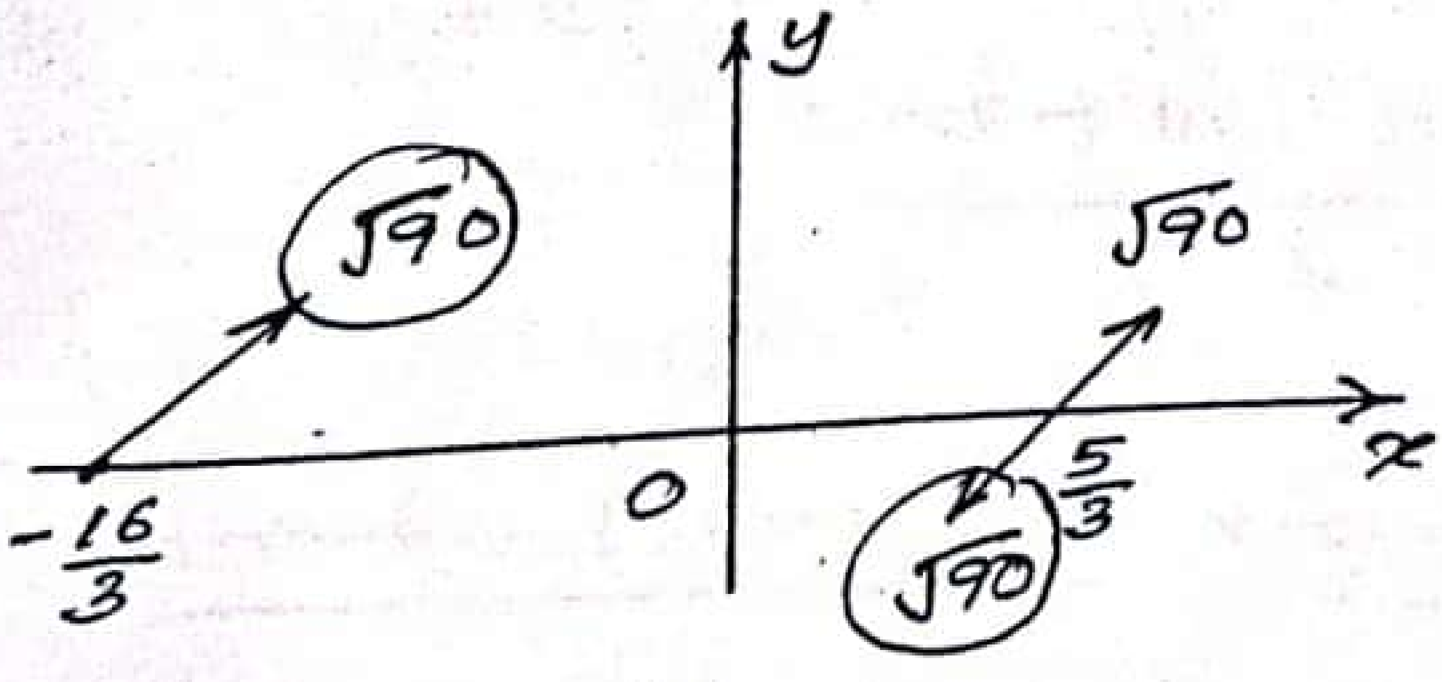


The eqⁿ of line of action

$$\frac{y-0}{x+\frac{16}{3}} = \frac{1}{3}$$ ⑤

$3x-9y+16=0$ $\boxed{25}$

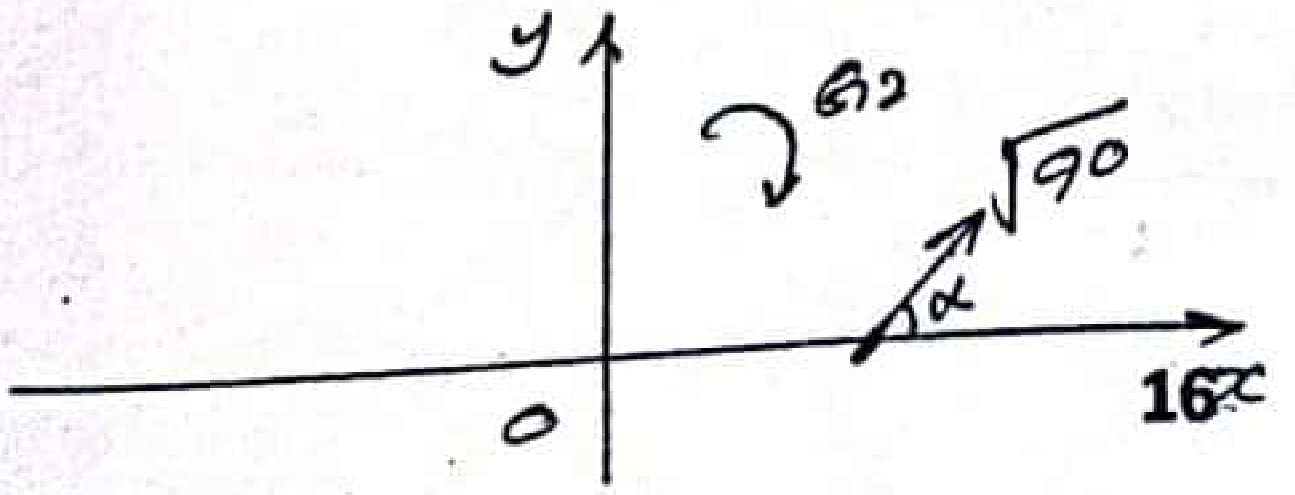
v)



$G_2 = \sqrt{90}\times\frac{21}{3}\sin\alpha$ ⑤

$= \sqrt{90}\times7\times\frac{1}{\sqrt{10}}$

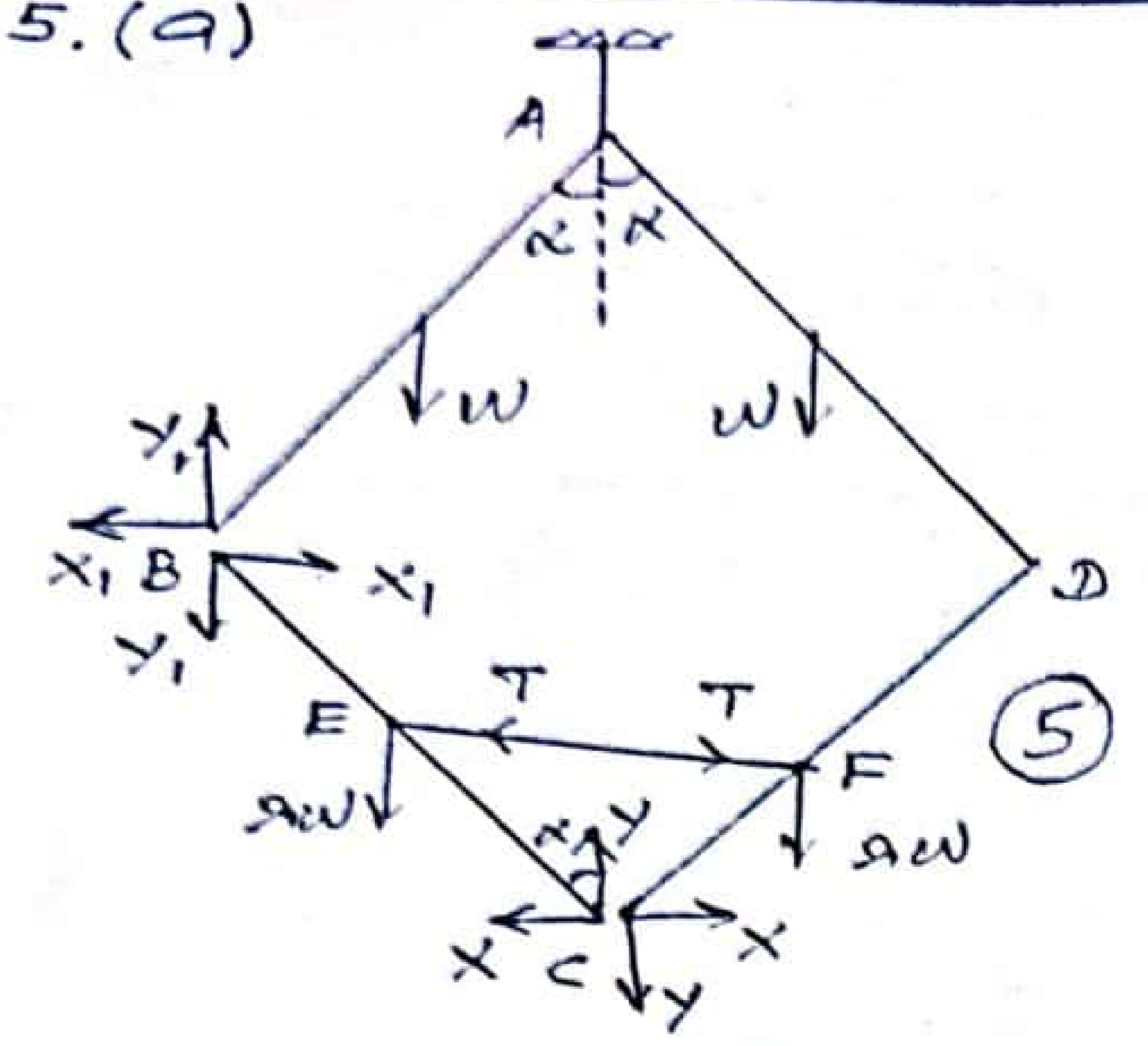
$= 21$ Nm ⑤



Apply a moment of a couple with magnitude 21 Nm in the anticlockwise dirⁿ ⑤

$\boxed{15}$

15. (a)



By symmetry $Y = 0$

For eq^m of the rod BC ⑤

$\leftarrow \quad X + T - X_1 = 0$ — ①

$\downarrow \quad Y_1 + \lambda w = 0$ ⑤

$Y_1 = -\lambda w$

taking moments about B ↻

$X \cdot 2a\cos\alpha + T \cdot a\cos\alpha + \lambda w\, a\sin\alpha = 0$ ⑩

$2X + T = -\lambda w\tan\alpha$ — ②

For eq^m of rod AB, taking moments about A ↻

$X_1 \cdot 2a\cos\alpha + Y_1 \cdot 2a\sin\alpha - w\, a\sin\alpha = 0$ ⑩

$2X_1 - 2\lambda w\tan\alpha - w\tan\alpha = 0$

$X_1 = \dfrac{(2\lambda+1)w\tan\alpha}{2}$ ⑤

② − ①

$X + X_1 = -\lambda w\tan\alpha$

$X = -\lambda w\tan\alpha - \dfrac{(2\lambda+1)w\tan\alpha}{2}$

$= -\dfrac{(4\lambda+1)w\tan\alpha}{2}$

① ⇒ $T = X_1 - X$

$= \dfrac{(2\lambda+1)w\tan\alpha}{2} + \dfrac{(4\lambda+1)w\tan\alpha}{2}$

$= \dfrac{(3\lambda+1)w\tan\alpha}{2}$

The magnitude of the reaction at the joint B

$$R_1 = \sqrt{X_1^2 + Y_1^2}$$

$$= \sqrt{\frac{(2a+1)^2 w^2 \tan^2\alpha}{4} + a^2 w^2}$$ ⑤

$$= \frac{w}{2}\sqrt{(2a+1)^2 \tan^2\alpha + 4a^2}$$

direction

$$\tan\beta = \frac{Y_1}{X_1}$$

$$= \frac{aw}{\frac{(2a+1)w\tan\alpha}{2}}$$ ⑤

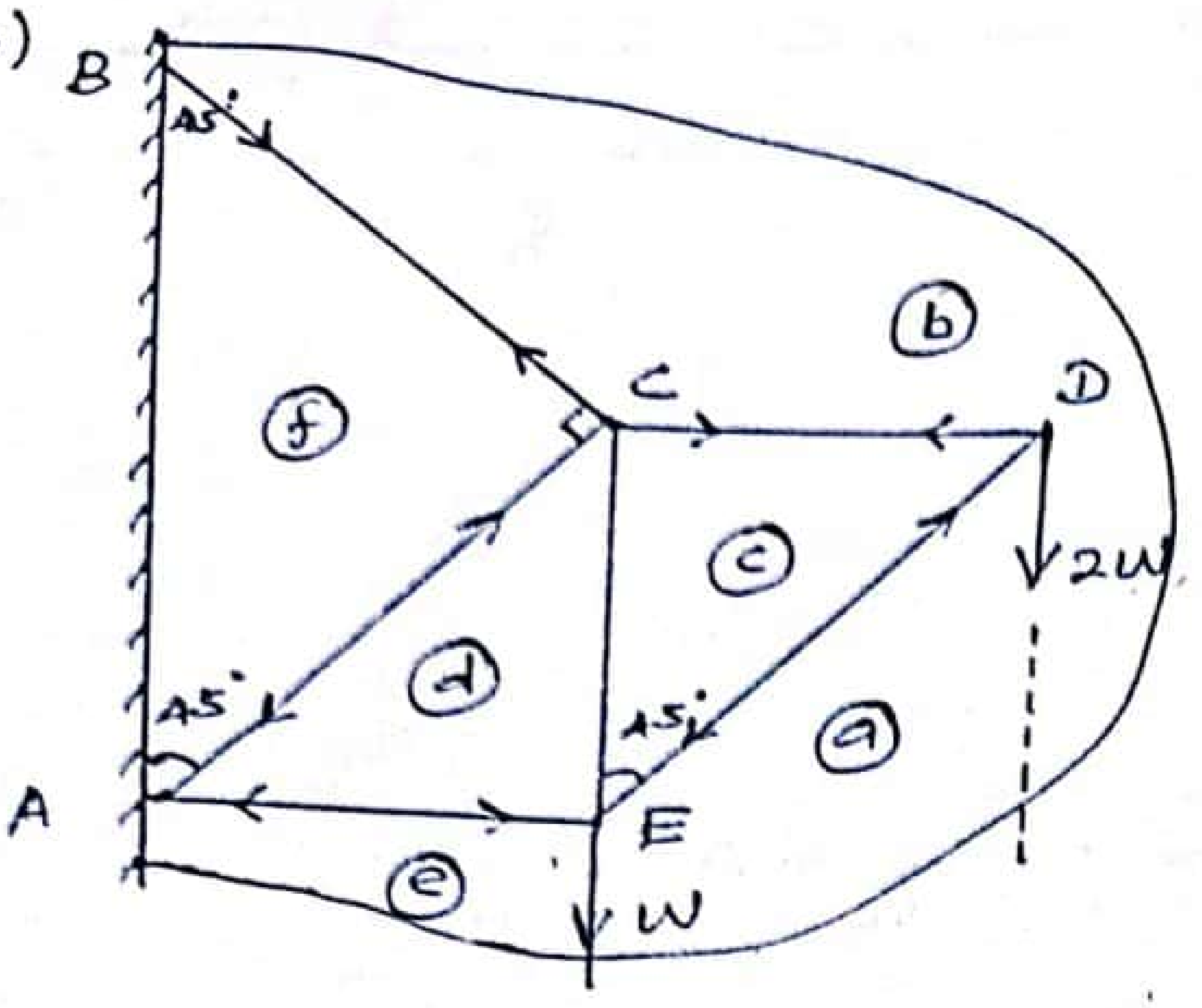
$$= \frac{2a}{2a+1} \cot\alpha$$

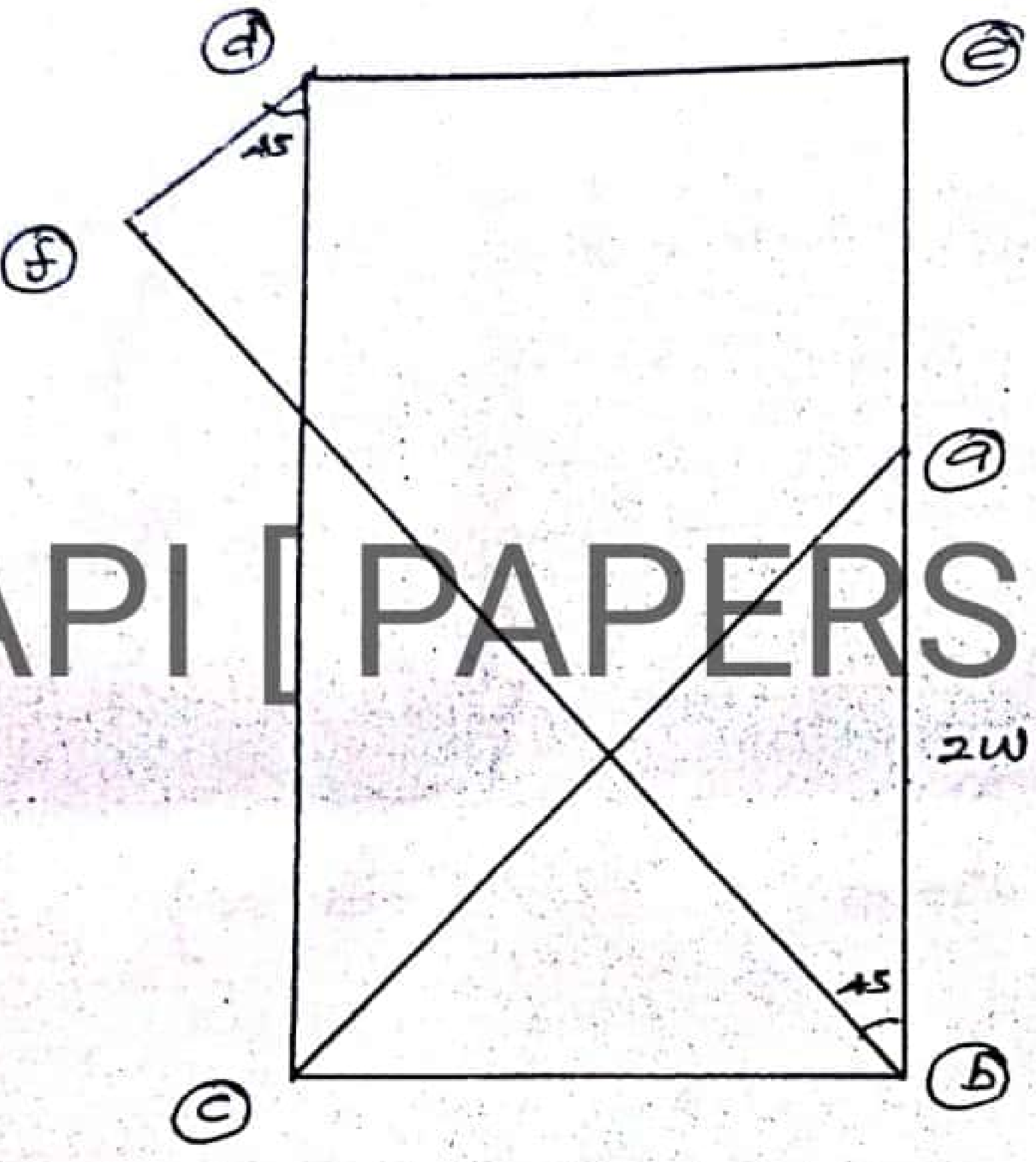
$$\beta = \tan^{-1}\left[\frac{2a}{2a+1} \cot\alpha\right]$$ ⑤

The magnitude of the reaction at the joint C is $\frac{(4a+1)w\tan\alpha}{2}$ ⑤ towards to the joint C.

$\boxed{60}$

(b)



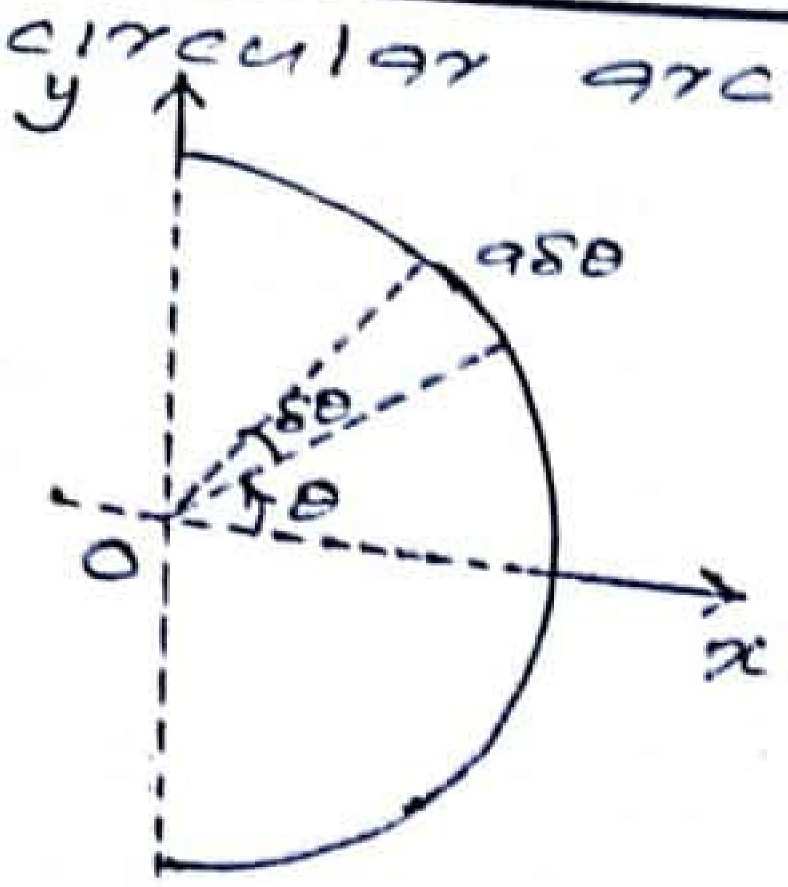


$\boxed{30}$

Rod	Stress	Nature	
AC	$fd = \frac{\sqrt{2}W}{2}$	Thrust	⑩
BC	$bf = \frac{5\sqrt{2}W}{2}$	Tension	⑩
CD	$bc = 2W$	Tension	⑩
CE	$cd = 3W$	Tension	⑩
AE	$de = 2W$	Thrust	⑩
DE	$ea = 2\sqrt{3}W$	Thrust	⑩

$\boxed{60}$

16.(i) For circular arc



By symmetry, the centre of mass lies on the x-axis ⑤

Taking moments about y-axis

$$\bar{x} = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i}$$

mass of the particle $m_i = a\delta\theta\rho$

where ρ mass per unit length

$$x_i = a\cos\theta$$

$$\bar{x} = \frac{\int_{-\frac{\pi}{2}}^{\pi/2} a\,d\theta\rho \, . \, a\cos\theta}{\int_{-\frac{\pi}{2}}^{\pi/2} a\,d\theta\rho}$$ ⑤

$$= a\frac{\int_{-\frac{\pi}{2}}^{\pi/2} \cos\theta\, d\theta}{\int_{-\frac{\pi}{2}}^{\pi/2} d\theta}$$

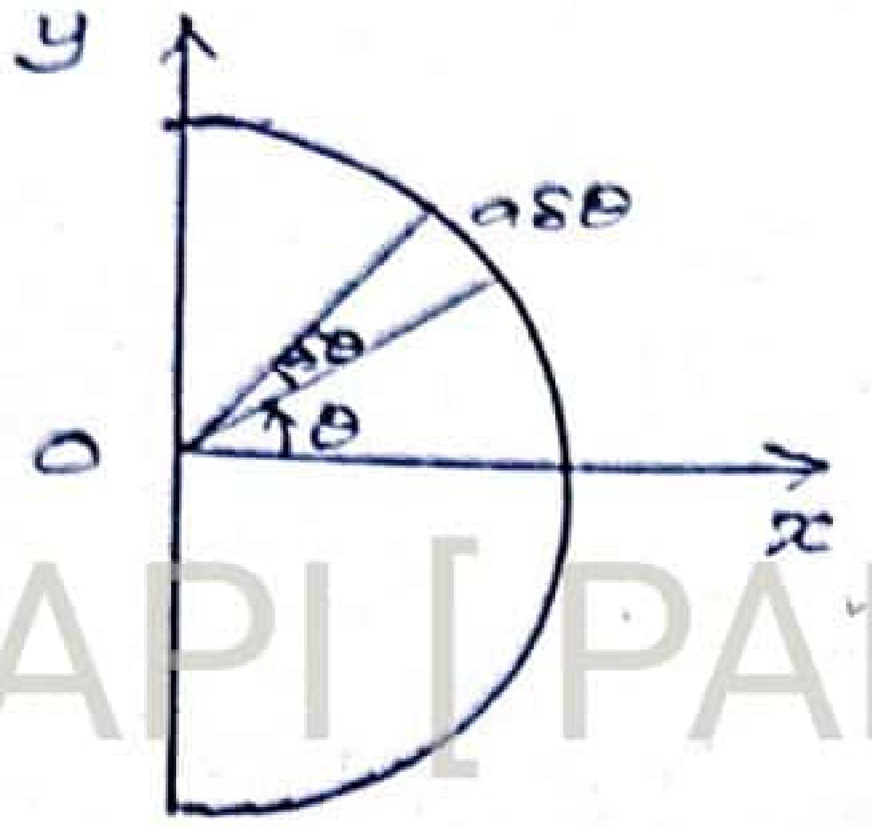
$$= a\frac{[\sin\theta]_{-\frac{\pi}{2}}^{\pi/2}}{[\theta]_{-\pi/2}^{\pi/2}}$$ ⑤

$$= a\frac{[\sin\frac{\pi}{2} - \sin(-\frac{\pi}{2})]}{\frac{\pi}{2} - (-\frac{\pi}{2})}$$

$$G \equiv \left(\frac{2a}{\pi}, 0\right)$$ $\boxed{20}$

$$= \frac{2a}{\pi}$$ ⑤

(ii) For circular laminar



By symmetry the centre of mass lies on the x- axis ⑤

Taking moments about y- axis

$$\bar{x} = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i}$$

mass of the particle $m_i = \frac{1}{2}a.ad\theta\rho$

$x_i = \frac{2a}{3}\cos\theta$

$$\bar{x} = \frac{\int_{-\frac{\pi}{2}}^{\pi/2} \frac{1}{2}a\,ad\theta\rho\frac{2a}{3}\cos\theta}{\int_{-\frac{\pi}{2}}^{\pi/2}\frac{1}{2}a\,ad\theta\rho}$$ ⑤

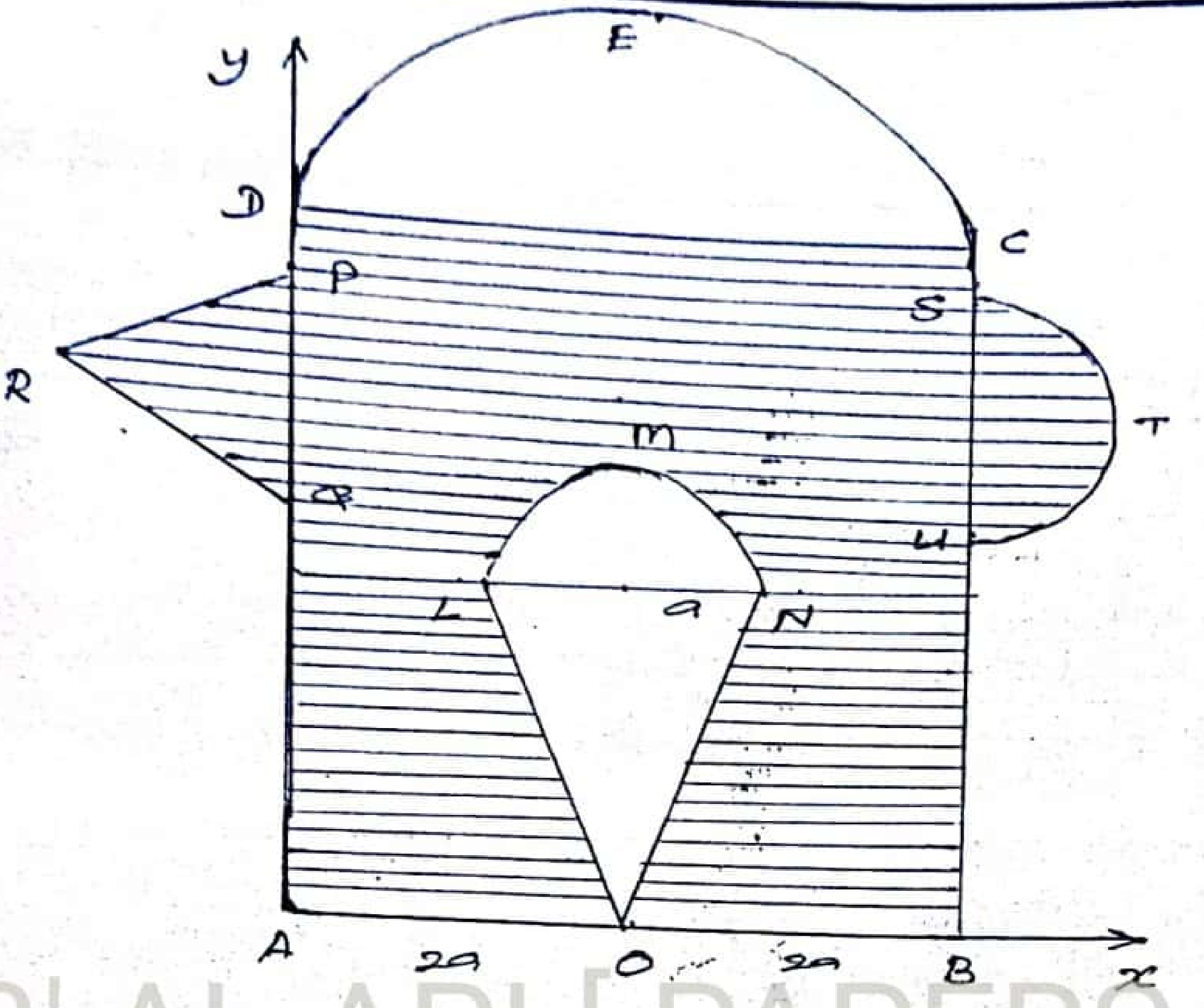
$$= \frac{2a}{3}\cdot\frac{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos\theta\,d\theta}{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}d\theta}$$

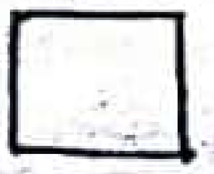
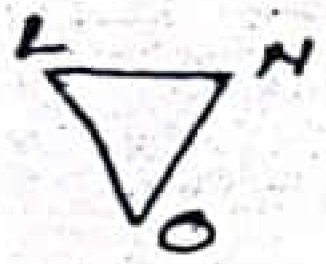
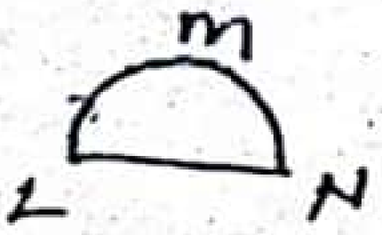
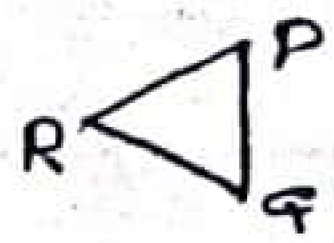
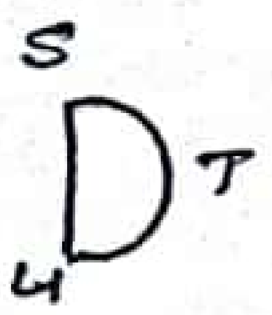
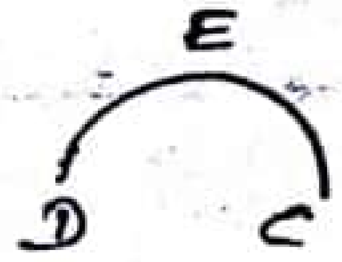
$$= \frac{2a}{3}\,\frac{[\sin\theta]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}}{\frac{\pi}{2}-(-\frac{\pi}{2})}$$ ⑤

$$= \frac{2a}{3}\,\frac{\sin\frac{\pi}{2}-\sin(-\frac{\pi}{2})}{\pi}$$

$$= \frac{4a}{3\pi}$$ ⑤ $G \equiv (\frac{4a}{3\pi}, 0)$

$\boxed{20}$



object	mass	distance from y-axis to c.m	distance from x-axis to c.m.	
 ABCD	$20a^2\sigma$ ⑤ $= 20M$	$2a$	$\frac{5a}{2}$	⑤
- 	$\frac{1}{2}\cdot 2a\cdot 2a\sigma$ ⑤ $= 2M$	$2a$	$\frac{2}{3}\,2a$	⑤
- 	$\frac{\pi a^2}{2}\sigma$ ⑤ $= \frac{\pi M}{2}$	$2a$	$2a+\frac{4a}{3\pi}$	⑤
	$\frac{1}{2}2a\,2a\,\sigma$ ⑤ $= 2M$	$-\frac{1}{3}\cdot 2a$	$\frac{7a}{2}$	⑤
	$\frac{\pi a^2}{2}\sigma$ ⑤ $= \frac{\pi M}{2}$	$4a+\frac{4a}{3\pi}$	$\frac{7a}{2}$	⑤
	$\pi\cdot 2a\frac{a\sigma}{2}$ ⑤ $= \pi M$	$2a$	$5a+\frac{2\cdot 2a}{\pi}$	⑤
⑤	$20M + 2\pi M$ $= M(\pi+20)$	$\bar{x}$	$\bar{y}$	

$\boxed{65}$

$$20m.2a - 2m.2a - \frac{\pi m}{2}.2a + 2m\left(-\frac{2a}{3}\right) + \frac{\pi m}{2}\left(4a + \frac{4a}{3\pi}\right)$$

$$+ \pi m.2a = m(\pi+20)\bar{x}$$ ⑩

$$40a - 4a - \pi a - \frac{4a}{3} + 2\pi a + \frac{2a}{3} + 2\pi a$$

$$\frac{106a}{3} + 3\pi a = (\pi+20)\bar{x}$$

$$\bar{x} = \frac{(9\pi+106)a}{3(\pi+20)}$$ ⑤

$$20m.\frac{5a}{2} - 2m.\frac{4a}{3} - \frac{\pi m}{2}\left(2a + \frac{4a}{3\pi}\right) + 2m\,\frac{7a}{2}$$ ⑩

$$+ \frac{\pi m}{2}.\frac{7a}{2} + \pi m\left(5a + \frac{4a}{\pi}\right) = m(\pi+20)$$ ⑤

$$50a - \frac{8a}{3} - \pi a - \frac{2a}{3} + 7a + \frac{7\pi a}{4} + 5\pi a + 4a$$

$$= (\pi+20)\bar{y}$$

$$(\pi+20)\bar{y} = \frac{173a}{3} + \frac{23\pi a}{4}$$

$$\bar{y} = \frac{(69\pi+692)a}{12(\pi+20)}$$ ⑤ $\boxed{35}$

Let AB makes an angle θ with the vert

$$\tan\theta = \frac{\bar{y}}{\bar{x}}$$ ⑤

$$= \frac{(69\pi+692)a}{12(\pi+20)} \cdot \frac{3(\pi+20)}{(9\pi+106)a}$$

$$= \frac{69\pi+692}{4(9\pi+106)}$$ ⑤ $\boxed{10}$

7.(a) Total Probability

If $\{A_1, A_2, \dots, A_n\}$ is a partition of the event space $\mathcal{A}$ corresponding to the sample space Ω of a certain random experiment, such that $P(A_i) > 0$ and if X is any event in the event space $\mathcal{A}$

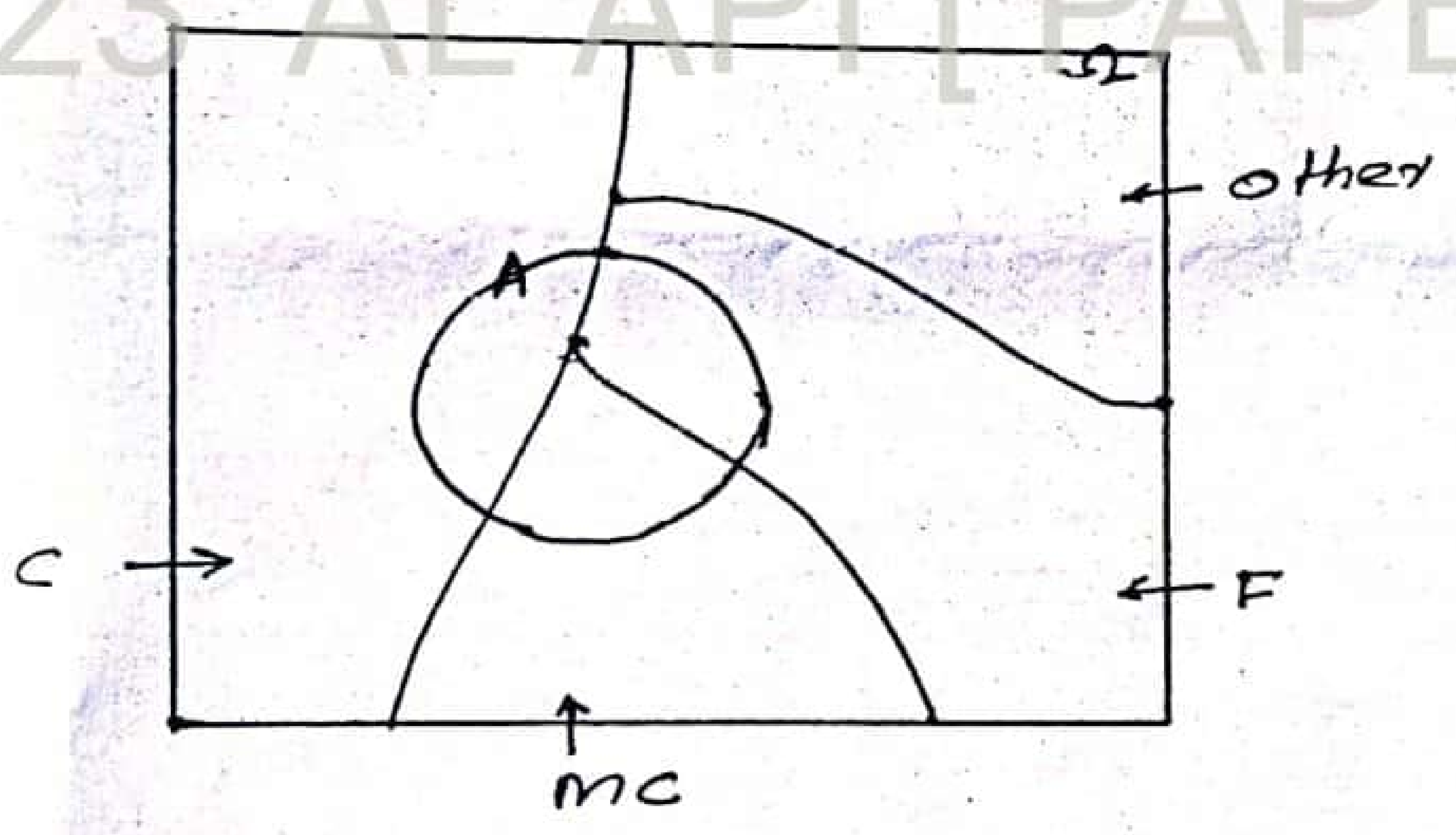
then $P(X) = \sum_{i=1}^{n} P(X|A_i)\, P(A_i)$. $\boxed{10}$

C : performs his journey by a car

MC : performs his journey by motor cycle

F : performs his journey on foot

A : facing an accident



$P(C) = \frac{1}{3}$

$P(MC) = \frac{1}{4}$

$P(F) = \frac{1}{5}$

$P(A|C) = \frac{1}{10}$

$P(A|MC) = \frac{2}{3}$

$P(A|F) = \frac{1}{20}$

The total probability

$$P(A) = \underset{⑤}{P(A|C)\,P(C)} + \underset{⑤}{P(A|MC)\,P(MC)} + \underset{⑤}{P(A|F)\,P(F)}$$

$$= \frac{1}{10}\cdot\frac{1}{3} + \frac{2}{3}\cdot\frac{1}{4} + \frac{1}{20}\cdot\frac{1}{5}$$

$$= \frac{1}{30} + \frac{1}{6} + \frac{1}{100}$$

$$= \frac{10 + 50 + 3}{300}$$

$$= \frac{63}{300}$$

$$= \frac{21}{100}$$ ⑤ $\boxed{20}$

(i) $P(C|A) = \dfrac{P(A|C)\,P(C)}{P(A)}$ ⑤

$= \dfrac{\frac{1}{10}\cdot\frac{1}{3}}{\frac{21}{100}}$

$= \dfrac{10}{63}$ ⑤ $\boxed{10}$

(ii) $P(F|A) = \dfrac{P(A|F)\,P(F)}{P(A)}$ ⑤

$= \dfrac{\frac{1}{20}\cdot\frac{1}{5}}{\frac{21}{100}}$

⑤ $\boxed{10}$

$= \dfrac{21}{100}$

(iii) $P(MC|A) = \dfrac{P(A|MC)\,P(MC)}{P(A)}$ ⑤

$= \dfrac{\frac{2}{3}\cdot\frac{1}{4}}{\frac{21}{100}}$

$= \dfrac{50}{63}$ ⑤ $\boxed{10}$

(b) $\{x_1, x_2, \ldots, x_n\}$

Mean $\bar{x} = \dfrac{x_1 + x_2 + \cdots + x_n}{n}$

$= \dfrac{\sum_{i=1}^{n} x_i}{n}$ ⑤

Variance $\sigma^2 = \dfrac{(x_1-\bar{x})^2 + (x_2-\bar{x})^2 + \cdots + (x_n-\bar{x})^2}{n}$

$= \dfrac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}$ ⑤ $\boxed{10}$

$u_i = \dfrac{x_i - A}{c} \Rightarrow x_i = cu_i + A$

$\bar{x} = \dfrac{\sum_{i=1}^{n}(cu_i + A)}{n}$ ⑤

$= c\dfrac{\sum_{i=1}^{n} u_i}{n} + \dfrac{\sum_{i=1}^{n} A}{n} = A + c\dfrac{\sum_{i=1}^{n} u_i}{n}$ ⑤ $\boxed{10}$

$$\sigma^2 = \frac{\Sigma (x_i - \bar{x})^2}{n}$$

$$= \frac{\Sigma (x_i^2 - 2\bar{x}\, x_i + \bar{x}^2)}{n}$$ ⑤

$$= \frac{\Sigma x_i^2}{n} - 2\bar{x}\frac{\Sigma x_i}{n} + \frac{\Sigma \bar{x}^2}{n}$$ ⑤

$$= \frac{\Sigma x_i^2}{n} - \bar{x}^2$$

$$= \frac{\Sigma (c u_i + A)^2}{n} - \left(A + c\frac{\Sigma u_i}{n}\right)^2$$ ⑤

$$= c^2\frac{\Sigma u_i^2}{n} + 2AC\frac{\Sigma u_i}{n} + A^2$$

⑤

$$- \left[A^2 + 2AC\frac{\Sigma u_i}{n} + c^2\left(\frac{\Sigma u_i}{n}\right)^2\right]$$

$$= c^2\left[\frac{\Sigma u_i^2}{n} - \left(\frac{\Sigma u_i}{n}\right)^2\right]$$ ⑤ $\boxed{25}$

⑤ ⑤ ⑤

class interval	Mid value x_i	Freq. f_i	deviation d_i	$u_i = \frac{d_i}{20}$	u_i^2	$f_i u_i$	$f_i u_i^2$
0 – 20	10	7	−40	−2	4	−14	28
20–40	30	16	−20	−1	1	−16	16
40–60	50	22	0	0	0	0	0
60–80	70	10	20	1	1	10	10
80–100	90	5	40	2	4	10	20
		60				−10	74

$\Sigma f_i u_i = -10$ ⑤

$\Sigma f_i u_i^2 = 74$ ⑤ $\boxed{25}$

Mean

$$\bar{x} = A + c\frac{\sum f_i u_i}{n}$$

$$= 50 + 20 \times \frac{(-10)}{60}$$ ⑤

$$= 50 - \frac{10}{3}$$

$$= \frac{140}{3}$$

$$= 46.66 \simeq 46.7$$ ⑤ $\boxed{10}$

Variance

$$\sigma^2 = c^2\left[\frac{\sum f_i u_i^2}{n} - \left(\frac{\sum f_i u_i}{n}\right)^2\right]$$

$$= 20^2\left[\frac{74}{60} - \left(-\frac{10}{60}\right)^2\right]$$ ⑤

$$= \frac{400}{60}\left[74 - \frac{100}{60}\right]$$

$$= \frac{20}{3}\left[74 - \frac{5}{3}\right]$$

$$= \frac{20}{3} \times \frac{217}{3}$$

$$= \frac{4340}{9}$$

$$= 482.2$$ ⑤ $\boxed{10}$